O'ZBEKISTON RESPUBLIKASI OLIY TA'LIM,
FAN VA INNOVATSIYALAR VAZIRLIGI

TERMIZ DAVLAT UNIVERSITETI

O'ZBEKISTON RESPUBLIKASI
FANLAR AKADEMIYASI

V.I.ROMANOVSKIY NOMIDAGI
MATEMATIKA INSTITUTI

ALGEBRA VA ANALIZNING DOLZARB MASALALARI

XALQARO ILMIY-AMALIY KONFERENSIYA

Termiz, 20-21oktabr 2025-yil

МЕЖДУНАРОДНАЯ НАУЧНО-ПРАКТИЧЕСКАЯ КОНФЕРЕНЦИЯ

АКТУАЛЬНЫЕ ПРОБЛЕМЫ АЛГЕБРЫ И АНАЛИЗА

Термез, 20–21 октября 2025 год

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МИНИСТЕРСТВО ВЫСШЕГО ОБРАЗОВАНИЯ, НАУКИ И ИННОВАЦИЙ РЕСПУБЛИКИ УЗБЕКИСТАН

ТЕРМЕЗСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН

СБОРНИК МАТЕРИАЛОВ

Международной научно-практической конференция по

АКТУАЛЬНЫМ ПРОБЛЕМАМ АЛГЕБРЫ И АНАЛИЗА

Термез, 20–21 октября 2025 года.

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MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATIONS OF THE REPUBLIC OF UZBEKISTAN

TERMEZ STATE UNIVERSITY

INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

ACADEMY OF SCIENCES OF THE REPUBLIC OF UZBEKISTAN

COLLECTION OF MATERIALS

International Scientific-Practical Conference on

CURRENT PROBLEMS OF ALGEBRA AND ANALYSIS

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Актуальные проблемы алгебры и анализа: сборник материалов Международной научно-практической конференции (20–21 октября 2025 года, г.Термез, Узбекистан). – Термез, 2025. -370с.

Данный сборник включает материалы Международной научной-практической конференции на тему «Актуальные проблемы алгебры и анализа», проведенной в Термезском государственном университете с целью расширения сотрудничества ученых, работающих в системе высших учебных заведений и научно-исследовательских институтах нашей республики, обсуждения новых результатов в области математики и методов ее преподавания, а также определения перспективных направлений. В частности, в него вошли доклады с пленарных заседаний, а также тезисы математиков, исследователей и магистрантов, ведущих научные исследований в области алгебры, анализа и смежных направлений на секционных заседаниях.

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$$+q^N\left(\frac{g_1h^2}{4}-\sqrt{3}\Big(d^+h-\frac{1}{2}g_1h^2N\Big)+3(1-\sqrt{3})f_2(N)\Big)\right);$$

$$C_\beta=\frac{6}{h^3}\Bigg(q^\beta\left(-\frac{1}{4}g_1h^2+\sqrt{3}d^-h+3(1-\sqrt{3})f_2(0)\right)+6\sqrt{3}\sum_{\gamma=0}^{N}q^{|\beta-\gamma|}f_2(\gamma)+f_2(\beta-1)-$$

$$-8f_2(\beta)+f_2(\beta+1)+q^{N-\beta}\left(\frac{1}{4}g_1h^2-\sqrt{3}\left(d^+h-\frac{1}{2}g_1h^2N\right)+3(1-\sqrt{3})f_2(N)\right)\Bigg);$$

$$C_N=\frac{6}{h^3}\Bigg(q^N\left(-\frac{1}{4}g_1h^2+\sqrt{3}d^-h+(3-3\sqrt{3})f_2(0)\right)+f_2(N-1)+6\sqrt{3}\sum_{\gamma=0}^{N}q^{N-\gamma}f_2(\gamma)+$$

$$+(1-\sqrt{3})\Big(d^+h-\frac{1}{2}g_1h^2N\Big)-(4+3\sqrt{3})f_2(N)\Bigg);$$

where $q=\sqrt{3}-2$ is the root of the second-order Euler polynomial.

References

1. **R. Estrada, R.P. Kanwal** *Singular Integral Equations.* BirkhΓ¤user, Boston, 2000.
2. **Sobolev S.L.** *Introduction to the theory of cubature formulas.* - Nauka, Moscow, 1974, 808 p.

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Analysis of the efficiency and complexity of parallel numerical solution algorithms in a model of radon volumetric activity with a fractional derivative of variable order

Tverdyi D. A.[1,2], Parovik R. I.[1]
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In modern science, close attention is paid to fundamental directions that have practical applications. the deformed state of the medium. For the Kamchatka region, in particular, this is a research of the processes of migration of subsurface radon gas using the most modern mathematical modeling methods in order to interpret anomalies preceding earthquakes. The authors in [1] propose mathematical hereditary α and $\alpha(t)$-models of RVA (volumetric radon activity), taking into account the nonlocality of the radon transport process over time. The main hypothesis of the hereditary models of OAR is that the order of the fractional derivative is associated with a change in the permeability of the geological environment due to a change in the stress-strain state of the medium.

In [2], the inverse problem is formulated and solved for the hereditary α-RVA model in order to restore the values of the model parameters, on the basis of which estimates of changes in the radon flux density are given with a change in the stress-strain state of the medium. The estimates obtained are reliable and more accurate than those based on the ODE model. The practical significance of the evaluation results is that it can help in more accurate selection of radon monitoring points.

When solving inverse problems, it becomes necessary to cyclically recalculate the direct problem for different values of model parameters and compare the results with experimental data. At the same time, a direct problem can have a sufficiently high computational complexity both when increasing the size of the input data N and when choosing a solution method. Therefore, it is important that the solution of each individual direct problem is completed in the shortest time, but at the same time it is necessary to take into account the possible memory costs of the algorithm used. All this leads us to the development of various parallel algorithms for the numerical solution of RVA model equations. This study examines parallel implementations of numerical solution methods: EFDS (non-local explicit finite difference scheme) [3] and IFDS-MNM (non-local implicit finite difference scheme solved by a modified Newton method) [3] in order to evaluate the computational complexity and efficiency of these algorithms. In particular, EFDS-omp and IFDS-MNM-omp based on the OpenMP API, as well as EFDS-hybrid and IFDS-MNM-hybrid hybrid CPU-GPU algorithms based on the OpenMP and CUDA APIs.

As a test example, we use hereditary $\alpha(t)$-RVA model, is a Cauchy problem of the form:

$$\partial_{0,t}^{\alpha(t)}\bar{A}(t) = -a(t)\bar{A}(t)^2 - b(t)\bar{A}(t) + c(t), \qquad \bar{A}(t) = \frac{A(t)}{A_{max}}, \quad \bar{A}(t_0) = \frac{A_0}{A_{max}},$$

where $\bar{A}(t)$ is RVA in dimensionless form; $A(t)$ –RVA; A_{max} is the maximum RVA value observed in the data; A_0 is the RVA at the initial moment of time; $t \in [t_0, T]$ is the time of the process under consideration; t_0 and $T > 0$ are the initial and final moments of time; $\partial_{0,t}^{\alpha(t)}\overline{A}(t) = \dfrac{1}{\Gamma(1-\alpha(t))}\displaystyle\int_0^t \dfrac{\dot{\overline{A}}(\tau)d\tau}{(t-\tau)^{\alpha(t)}}$– is a member of the model describing the delay in radon transport through the medium, a fractional derivative of the Gerasimov-Caputo type [4] of variable order $0 < \alpha(t) < 1$. Since only the time from the beginning to the end of the model calculation is important for this study, we will not specify the dimensions of the model members further to simplify the work.

According to [1], the problem is considered at the values of the parameters: $T = 22, \overline{A} = 0, \overline{A}_{max} = 1, \lambda_0 = 0.05,$ $\quad a(t) = -2\lambda_0 + 12\lambda_0\left(2\cos\left(\frac{\pi\ t}{T}\right)^2 + \cos\left(\frac{2\pi\ t}{T} - \frac{\pi}{11}\right)^2\right),$ $\quad b(t) = \lambda_0,$ $\alpha(t) = 0.99\left(1 - \left(\frac{(T-t)}{T}\cos\left(\frac{3\pi\ t}{T}\right)^2\right)\right),$ $\quad c(t) = 12\lambda_0\left(\frac{12(T-t)}{10\ T}\sin\left(\frac{2\pi\ t}{T}\right)^2 + \frac{(T-t)}{T}\sin\left(\frac{3\pi\ t}{T}\right)^2\right),$ but with different N multiples of 100, this is done for clarity, simplification of plotting and convenience of working with estimates. This test case was chosen for two reasons. Firstly, such a task is not synthesized specifically to analyze the effectiveness of an algorithm, but is important from the point of view of applied mathematics. Secondly, the task may have a sufficiently high computational complexity for efficiency research with an increase of N by several tens of times.

The Cauchy problem was solved numerically, but regardless of the scheme, the solution is considered in a uniform grid domain: $\widehat{\Omega} = \{(t_i = ih) : 0 \le i < N\}, h = T/N, \quad \overline{A}(t) = \overline{A}_i, 0 < \overline{A}_i < 1, \quad a(t) = a_i, c(t) = c_i, \alpha(t) = \alpha_i$.

We will analyze the efficiency and estimates of the "time complexity"of parallel algorithms based on data on the average execution time of $T_{p,g}(N)$ in [sec.], where p is the number of CPU threads; g is the number of GPU multiprocessors; N is the size of the input data. Similarly, notation is introduced for estimates of "memory complexity"of $M_{p,g}^{C}(N)$ – RAM CPU and $M_{p,g}^{G}(N)$ – RAM GPU. The terms "number of operations"and "computational complexity"are sometimes used synonymously with the concept of "algorithm execution time"[5].

To analyze the efficiency, a series of computational experiments of the test example was conducted at $N = 6000$, but the number of CPU threads used changed in increments of 2 threads. Next, the data on $T_{p,g}(N)$ is considered in terms (TAECO) applicable to algorithms: T execution time, A acceleration, E efficiency, C cost, O cost-optimal indicator.

The complexity estimates were carried out as follows: a series of computational experiments of the test example was conducted at $N = 1000...15,000$ and a fixed, optimal number (16 pcs.) of CPU threads. Next, using polynomial interpolation, we construct $\mathbb{T}$, the algorithm's execution time function, where $\mathbb{T}$ is a n–th degree polynomial, and the growth order of the execution time function is determined by its senior, dominant term $f_{p,g}(N)$. We give Θ-asymptotically accurate estimates of the complexity of algorithms when the condition is fulfilled:

$$\Theta(f_{p,g}(N)) = \left\{ \mathbb{T}_{p,g}(N) : \begin{array}{c} \exists k_1 > 0, k_2 > 0, \quad \exists N_0 \in \{0,1,2,...\}, \\ 0 \le k_1 f_{p,g}(N) \le \mathbb{T}_{p,g}(N) \le k_2 f_{p,g}(N), \qquad \forall N \ge N_0. \end{array} \right\}$$

A similar logic applies to «memory complexity» and $\mathbb{M}$ are functions of the algorithm's memory usage.

Computational experiments on the test example were carried out using the FEVO v1.0 software package, designed to simulate and analyze the volumetric activity of radon as a precursor to strong earthquakes in Kamchatka. The MATLAB environment was used to analyze efficiency, construct polynomials, and estimate complexity. All calculations were carried out on a computer with the following characteristics: CPU – AMD Ryzen 9 7950X, 16 × 4.5 GHz core, cache L2 16 Mb & L3 64 Mb; RAM вЂ 96 Gb; GPU – GeForce RTX 4090, 24 Gb, 2235 MHz, ALU 16384.

As a result, the following conclusions can be drawn for the considered software implementations of numerical methods for solving the hereditary model equation of radon volume activity:

- An analysis of the efficiency and optimal CPU utilization showed that for all the considered parallel algorithms, an increase in the number of CPU threads of more than 16 does not give a significant increase in performance;

- It is shown that the parallel EFDS-omp and EFDS-hybrid algorithms do not provide a significant increase in computing speed (on the order of 30%) compared to EFDS. However, taking into account the stability constraints associated with using an explicit scheme and the fact that orders of magnitude more RAM is needed, their use can speed up calculations when solving inverse problems;

- At the same time, the parallel IFDS-MNM-omp and IFDS-MNM-hybrid algorithms provide a significant increase in computing acceleration by 13 and 17 times, respectively, while increasing RAM costs by no more than 2.5 and 5 times, respectively, compared with sequential IFDS-MNM;

- The parallel EFDS-omp and EFDS-hybrid algorithms have an asymptotically accurate time complexity estimate of the order of $\Theta(n)$, however, the RAM estimate is already on the order of $\Theta(n^2)$;

- The parallel algorithms IFDS-MNM-omp and IFDS-MNM-hybrid have asymptotically accu-rate time and RAM complexity estimates of the order of $\Theta(n^2)$;

- It is also seen that when solving a test problem with a uniformly increasing input data size of N and an optimal number of CPU threads of 16, using `-hybrid` algorithms gives a significant advantage over `-omp` algorithms only when solving problems with $N \geq 15,000$, but with the total memory consumption of computing nodes, it is 4 times more. This is due to the fact that operations on vectors and matrices are carried out mainly on the GPU, which have an advantage over the CPU when working with large-dimensional tensors.

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References

1. **Tverdyi D.A., Makarov E.O., Parovik R.I.** *Hereditary Mathematical Model of the Dynamics of Radon Accumulation in the Accumulation Chamber.* Mathematics. - 2023. - vol. 11. - DOI: 10.3390/math11040850
2. **Tverdyi D.A., Makarov E.O., Parovik R.I.** *Estimation of radon flux density changes in temporal vicinity of the Shipunskoe earthquake with MW = 7.0, 17 August 2024 with the use of the hereditary mathematical model.* Geosciences. - 2025. - vol. 15. - DOI: 10.3390/geosciences15010030
3. **Tverdyi D.A., Parovik R.I.** *Investigation of Finite-Difference Schemes for the Numerical Solution of a Fractional Nonlinear Equation.* Fractal and Fractional. - 2022. - vol. 6. - DOI: 10.3390/fractalfract6010023
4. **Caputo M., Fabrizio M.** *On the notion of fractional derivative and applications to the hysteresis phenomena.* Meccanica. - 2017. - vol. 52. - DOI: 10.1007/s11012-017-0652-y
5. **Storer J.A.** *An Introduction to Data Structures and Algorithms.* Boston: BirkhГ¤user Boston. - 2012. - 599 p.

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SOLVING THE BURGERS-HUXLEY EQUATION BY USING THE SEMI-ANALYTICAL METHOD

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The Burgers-Huxley equation models were implemented to traffic flows, acoustics, turbulence theory, hydrodynamics and mechanics. Thus, in this note semi-analytical technique is applied to obtain a universal solution for Dirichlet and symmetric boundary conditions of Burgers-Huxley equation. One of the famous semi-analytical approximation method is Adomian decomposition method (ADM) and homotopy analysis method (HAM). The Adomian decomposition method was first introduced by George Adomian in 1980 to solve the system of stochastic equations. This decomposition method can be an effective procedure for obtaining analytical solutions without linearization or perturbation or restrictive assumptions on stochastic cases. The HAM is proposed by the Liao in 1992 as semi-analytical method for highly nonlinear problems. Unlike perturbation techniques, the HAM is independent of any small/large physical parameters at all and can always transfer a nonlinear problem into sequence of iterations which can be solved easily.

In this study, we examine the generalized Burgers-Huxley equation of the form

$$u_t(x,t) + \alpha u^{\delta}(x,t) u_x(x,t) - \omega u_{xx}(x,t)$$

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