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MATHEMATICS

MSC 47F52+47F05

ON THE STABILITY OF THE BOUNDARY VALUE PROBLEM FOR EVEN ORDER EQUATION

A.V. Yuldasheva

National University of Uzbekistan by Mirzo Ulugbeka, 100174, Uzbekistan, Tashkent c., VUZ gorodok st.
E-mail: yuasv86@mail.ru

In this paper we consider ill-posed problem for one even-order equation. The stability of the problem is proved with the additional assumption.

Key words: partial differential equations, ill-posed problem, boundary value problem, algebraic numbers, the simple continued fraction

Introduction

We consider following problem for even order equation:

\[ \frac{\partial^{2k}u}{\partial x^{2k}} - \frac{\partial^{2p}u}{\partial t^{2p}} = 0, \quad k, p \in N, \quad 0 < x < \pi, \quad 0 < t < \alpha\pi, \]

\[ \frac{\partial^{2m}u}{\partial x^{2m}}(0,t) = \frac{\partial^{2m}u}{\partial x^{2m}}(\pi,t) = 0, \quad m = 0, 1, ..., k - 1, \quad 0 \leq t \leq \alpha\pi, \]

\[ \frac{\partial^j u}{\partial t^j}(x,0) = \varphi_j(x), \quad j = 0, 1, ..., p - 1, \quad 0 \leq x \leq \pi, \]

\[ \frac{\partial^j u}{\partial t^j}(x,\alpha\pi) = \psi_j(x), \quad j = 0, 1, ..., p - 1, \quad 0 \leq x \leq \pi, \]

where \( \alpha \) is a positive constant.

If \( k = p = 1 \) we get The Dirichlet problem for the vibrating string equation, which is a classical ill-posed problem due to its irregular behavior. Its solution may neither exists, nor be uniquely determined, nor depend continuously on the data (see [1]-[3]).

In [2], the Dirichlet problem for the wave equation was studied with the additional assumption of an “a priori” bound for the gradient of the solution. Case when \( p = 1, k \in N \) was studied in [6].

The present research leads to some problems of Diophantine approximation.

Let us note that formulate problem is ill-posed problem if \( k - p \) is even number.

Therefore, the above problem cannot be suitably dealt with if \( \alpha \) and \( \varphi_j(x), \psi_j(x), j = 0, 1, ..., p - 1 \) are known within a certain approximation.

Main results and comments

Let \( \varphi_j(x), \psi_j(x), (j = 0, 1, \ldots, p - 1) \) be functions in \( C^{2k}[0, \pi] \) such that \( \varphi_j^{(2j)}(0) = \psi_j^{(2j)}(0) = 0, i = 0, 1, \ldots, k - 1, j = 0, 1, \ldots, p - 1 \).

Let \( t, E, \alpha, \delta \) be positive constants. We consider solutions \( u \) in \( C_{x,t}^{2k,2p}([0, \pi] \times (0, +\infty)) \) of the following problem:

\[
\frac{\partial^{2k}u}{\partial x^{2k}} - \frac{\partial^{2p}u}{\partial t^{2p}} = 0, \quad k, p \in \mathbb{N}, \quad 0 < x < \pi, \quad t > 0, \tag{1}
\]

\[
\frac{\partial^{2m}u}{\partial x^{2m}}(0,t) = \frac{\partial^{2m}u}{\partial x^{2m}}(\pi,t) = 0, \quad m = 0, 1, \ldots, k - 1, \quad t \geq 0, \tag{2}
\]

\[
\left\| \frac{\partial^j u}{\partial t^j}(x,0) - \varphi_j(x) \right\|_{L_2[0,\pi]} \leq \delta \pi \sqrt{E}, \quad j = 0, 1, \ldots, p - 1, \tag{3}
\]

\[
\left\| \frac{\partial^j u}{\partial t^j}(x,\tau_j \pi) - \psi_j(x) \right\|_{L_2[0,\pi]} \leq \delta \pi \sqrt{E}, \quad j = 0, 1, \ldots, p - 1, \quad |\tau_j - \alpha| \leq \delta, \tag{4}
\]

\[
\int_0^\pi \left( \left( \frac{\partial^k u}{\partial x^k} \right)^2 + \left( \frac{\partial^p u}{\partial t^p} \right)^2 \right) dx \leq E, \quad t \geq 0. \tag{5}
\]

for real numbers \( \tau_j, j = 0, 1, \ldots, p - 1 \) depending on \( u \) and satisfying \( |\tau_j - \alpha| \leq \delta \). The meaning \( |\tau_j - \alpha| \leq \delta \) is that the final time \( \alpha \pi \) is known up to a given error.

We denote by \( \Gamma_\delta \) the set of all \( C_{x,t}^{2k,2p}([0, \pi] \times (0, +\infty)) \) solutions of (1)-(5). We note that if \( \delta = 0 \), then the problem (1)-(5) is reduced to the classical boundary value problem with additional assumption (5). It was studied in [6] that this problem may have no solutions.

Let \( Diam_{\Gamma_\delta} = \sup_{v,w \in \Gamma_\delta} \| v - w \| \). Let \( v_1, v_2 \in \Gamma_\delta \). Then there are \( \tau_{ij} \) such that

\[
\left\| \frac{\partial^j u}{\partial t^j}(x,\tau_{ij} \pi) - \psi_j(x) \right\|_{L_2[0,\pi]} \leq \delta \pi \sqrt{E}, \quad i = 0, 1, \quad j = 0, 1, \ldots, p - 1, \quad |\tau_{ij} - \alpha| \leq \delta
\]

and let

\[
u(x,t) = v_1(x,t) - v_2(x,t), (x,t) \in [0, \pi] \times [0, +\infty) \tag{6}\]

Then \( u \in C_{x,t}^{2k,2p}([0, \pi] \times [0, +\infty)) \). Moreover, \( u \) satisfies equation (1), conditions (2) and following

\[
\left\| \frac{\partial^j u}{\partial t^j}(x,0) \right\|_{L_2[0,\pi]} \leq 2\delta \pi \sqrt{E}, \quad j = 0, 1, \ldots, p - 1, \tag{7}
\]

\[
\left\| \frac{\partial^j u}{\partial t^j}(x,\alpha \pi) - \psi_j(x) \right\|_{L_2[0,\pi]} \leq 4\delta \pi \sqrt{E}, \quad j = 0, 1, \ldots, p - 1, \tag{8}
\]

\[
\int_0^\pi \left( \left( \frac{\partial^k u}{\partial x^k} \right)^2 + \left( \frac{\partial^p u}{\partial t^p} \right)^2 \right) dx \leq 4E, \quad t \geq 0. \tag{9}
\]
It is easy to verify (1), (2), (7)-(9). We can write the function satisfying (1) and (2) in the following form

\[ u(x, t) = \sum_{n \geq 1} \sin n x \left\{ \frac{A_n \sin n^\frac{1}{2} (\alpha \pi - t)}{\sin n^\frac{1}{2} \alpha \pi} + B_n \sin n^\frac{1}{2} t \right\}. \]

Similarly, we can rewrite (7)-(9) as follows:

\[ \sum_{n \geq 1} A_n^2 \leq 8 \delta^2 \pi E, \quad \sum_{n \geq 1} B_n \sin^2 n^\frac{1}{2} \alpha \pi \leq 32 \delta^2 E, \]

\[ \left\| \frac{\partial^k u}{\partial x^k} (\cdot, t) \right\|_{L^2[0, \pi]}^2 + \left\| \frac{\partial^p u}{\partial t^p} (\cdot, t) \right\|_{L^2[0, \pi]}^2 \leq 4E, \ t \geq 0. \]

Defining:

\[ \sigma_n = \sqrt{\frac{\pi}{2}} \left( \frac{A_n \sin n^\frac{1}{2} (\alpha \pi - t)}{\sin n^\frac{1}{2} \alpha \pi} + B_n \sin n^\frac{1}{2} t \right), \]

we obtain from (12)

\[ \left\| \frac{\partial^k u}{\partial x^k} (\cdot, t) \right\|_{L^2[0, \pi]}^2 \leq \sum_{n \geq 1} n^{2k} \sigma_n^2 \leq 4E, \]

whence

\[ \| u (\cdot, t) \|_{L^2[0, \pi]}^2 = \sum_{n=1}^{N} \sigma_n^2 + \sum_{n=N+1}^{\infty} \sigma_n^2 < \sum_{n=1}^{N} \sigma_n^2 + \frac{4E}{N^{2k}}. \]

We now have following bound:

\[ \| u (\cdot, t) \|_{L^2[0, \pi]}^2 < \frac{\pi^2}{2} \max_{n=1, N} \left( \sin n^\frac{1}{2} \alpha \pi \right)^{-2} \sum_{n=1}^{N} \left[ A_n \sin^2 n^\frac{1}{2} (\alpha \pi - t) + B_n \sin^2 n^\frac{1}{2} \alpha \pi \cdot \sin^2 n^\frac{1}{2} t + 2 |A_n| |B_n| \sin n^\frac{1}{2} (\alpha \pi - t) \left| \sin n^\frac{1}{2} t \right| \left| \sin n^\frac{1}{2} \alpha \pi \right| \right] + \frac{4E}{N^{2k}}. \]

Therefore, from (10) and (11) it follows that

\[ \max_{t \in [0, \alpha \pi]} \| u (\cdot, t) \|_{L^2[0, \pi]}^2 = \| u \|^2 < 40 \delta^2 \pi^2 E \max_{n=1, N} \left( \sin n^\frac{1}{2} \alpha \pi \right)^{-2} + \frac{4E}{N^{2k}}. \]

Let

\[ \alpha = \frac{1}{a_1 + \frac{1}{a_2 + \ldots}} \]

be the simple continued fraction for \( \alpha \), where the partial quotients \( a_n \) are integers such that, \( a_n \geq 1 \).

We consider the set of irrational numbers with bounded partial quotients, i.e. the numbers \( \alpha \), for which there exists a constant \( A_\alpha \) satisfying \( a_n \leq A_\alpha \) for all \( n \). We note that if \( \alpha \) is a quadratic irrational, then the expansion of \( \alpha \) as a simple continued fraction is ultimately periodic, which implies that \( a_n \) has bounded partial quotients.
Then from theory of continued fractions (see [5] p.37) we easily obtain

$$\max_{n=1,N} \left( \sin \frac{k}{n} \alpha \pi \right)^{-2} < \left( \sin \frac{\pi}{(A\alpha+2)N^{1/2}} \right)^{-2}, N = 1, 2, \ldots.$$  

Since \( \sin x \geq \frac{3\sqrt{3}}{2\pi} x \) for \( x \in [0, \pi/3] \), we have for every \( N \)

$$\|u\|^2 < \frac{160}{27} \delta^2 \pi^2 E(A\alpha+2)^2 t \frac{2k}{P} + 4E t^{-2k}, N = 1, 2, \ldots.$$  

(13)

Now let

$$g(t) = \frac{160}{27} \delta^2 \pi^2 E(A\alpha+2)^2 t \frac{2k}{P} + 4E t^{-2k}.$$  

The minimum value of \( g \) for \( t > 0 \) is attained at

$$\bar{t} = \left( \frac{27P}{40} \right) \frac{2k(p+1)}{k(p+1)} \frac{d\pi(A\alpha+2)}{d\pi(A\alpha+2)}.$$  

Since \( g \) is an increasing function on the interval \([\bar{t}, +\infty)\), we have

$$g(\bar{t}+1) < g(\bar{t}+1).$$

We obtain

$$\|u\|^2 \leq \frac{160E}{27} (\delta \pi(A\alpha+2))^{2/p} \left[ 1 + \left( \frac{27P}{40} \right)^{2/p} + (\delta \pi(A\alpha+2))^{2/p} \right]^{2k}.$$  

(14)

So we proved following

**Theorem 1.** Let \( \alpha \) is an irrational number and has the simple continued fraction with bounded partial quotients. Then for \((\text{Diam} \Gamma_{\delta})^2 \) (14) is valid.

Now we use some results obtained in [4]. By corollary 6 of [7], since \( \alpha \) has a type \( \Omega < \infty \), there exist \( K = K(\theta, \alpha) > 0 \) and, for any \( \delta > 0 \), a number \( \xi \in R \setminus Q \) such that

$$|\xi - \alpha| < \delta,$$  

(15)

and

$$\max_{n=1,N} \left( \sin n\pi \xi \right)^{-2} \leq \left( \sin \left( \frac{\pi (3 - \sqrt{5})}{2N} \right) \right)^{-2}.$$  

(16)

for all \( N \geq K\delta^{-\theta} \). From (15) it follows that \( |\tau_j - \alpha| \leq \delta \), for every \( \tau_j \) satisfying \( |\xi - \tau_j| \leq 2\delta \).
If $u$ is defined by (6), we obtain from (8)

$$\left\| \frac{\partial^j u}{\partial t^j} (x, \xi \pi) \right\|_{L^2[0, \pi]} \leq 4 \delta \pi \sqrt{E}, \ j = 0, 1, \ldots, p - 1. \quad (17)$$

Therefore, $u$ satisfies conditions (1), (2), (7), (9) and (17). The solutions of the problem (1), (2), (7), (9) and (17) $u \in C_{x, t}^{2k, 2p} ([0, \pi] \times [0, +\infty))$ of the form

$$u(x, t) = \sum_{n \geq 1} \sin n x \left\{ A_n \sin \frac{\tilde{\pi} n (\xi \pi - t)}{\sin \frac{\tilde{\pi} n \xi \pi}{\tilde{\pi}} + B_n \sin \frac{n \tilde{\pi} t}{\tilde{\pi}} \right\},$$

which satisfies (10), (12) and

$$\sum_{n \geq 1} B_n \sin^2 \frac{n \tilde{\pi} \xi \pi}{\tilde{\pi}} \leq 72 \delta^2 E. \quad (18)$$

As in proof of theorem 1 we obtain

$$\|u\|^2 < 80 \delta^2 \pi^2 E \max_{n=1, N} \left( \sin \frac{n \tilde{\pi} \xi \pi}{\tilde{\pi}} \right)^{-2} + \frac{4E}{N^{2k}}, \ N = 1, 2, \ldots. \quad (19)$$

Using (16) and $\sin x \geq \frac{2}{\pi} x$ for all $x \in [0, \frac{\pi}{2}]$, we obtain

$$\|u\|^2 < \frac{80}{(3 - \sqrt{5})^2} \delta^2 \pi^2 E \left\{ \frac{2k}{N^{2k}} \right\} + \frac{4E}{N^{2k}}, \ N \geq K \delta^{-\theta}. \quad (19)$$

Let

$$g(t) = \frac{80}{(3 - \sqrt{5})^2} \delta^2 \pi^2 E t^p + \frac{4E}{t^{2k}}, \ t > 0. \quad (20)$$

The minimum of $g$ for $t > 0$ is attained at

$$\bar{t} = \left( \frac{p}{20} \right)^{2k(p+1)} \left( \frac{3 - \sqrt{5}}{\pi \delta} \right)^{p} \left( \frac{k(p+1)}{k(p+1) - p} \right).$$

We choose $\delta$ as

$$0 < \delta < \left\{ K \left( \frac{p}{20} \right)^{2k(p+1)} \left( \frac{3 - \sqrt{5}}{\pi \delta} \right)^{p} \right\}^{k(p+1)} \left( \frac{k(p+1)}{k\theta(p+1) - p} \right). \quad (21)$$

It follows from (21) that $\bar{t} < K \delta^{-\theta}$. Let $\bar{N}$ be the integer $\geq K \delta^{-\theta}$ for which the right side of (20) is minimum. Since $g$ is increasing on the interval $[\bar{t}, +\infty)$, $\bar{N}$ satisfies $K \delta^{-\theta} \leq \bar{N} < K \delta^{-\theta} + 1$. Hence

$$\|u\|^2 \leq g \left( K \delta^{-\theta} + 1 \right).$$
and finally
\begin{equation}
\|u\|^2 \leq \frac{80\pi^2E}{(3-\sqrt{5})^2} \left[ K\delta^{\frac{k}{2}} + \delta^p \right] \frac{2k}{p} + \frac{4E\delta^{2k}}{K^{2k}} \tag{22}
\end{equation}

which proofs following:

**Theorem 2.** Let \( \alpha \) be an irrational number and has a type \( \Omega < \infty \). Then for any fixed \( \theta, \frac{\Omega}{\Omega + 1} < \theta < 1 \), there is constant \( K = K(\theta, \alpha) > 0 \) such that

\begin{equation}
\|u\|^2 \leq \frac{80\pi^2E}{(3-\sqrt{5})^2} \left[ \frac{k}{K\delta^{\frac{k}{2}} + \delta^p} \right] \frac{2k}{p} + \frac{4E\delta^{2k}}{K^{2k}}
\end{equation}

for any \( 0 < \delta < \left\{ \left( K\left( \frac{p}{20} \right) \right)^{\frac{p}{2k(p+1)}} \left( \frac{3-\sqrt{5}}{\pi\delta} \right)^{\frac{p}{k(p+1)}} \right\}^{\frac{k(p+1)}{k\theta(p+1) - p}} \).

We conclude with the proof of the following:

**Theorem 3.** The problem (1)-(5) is stable if and only if \( \alpha \) is irrational. Moreover, if \( \alpha \) is irrational then \( \lim_{\delta \to 0} (\text{Diam} \Gamma_\delta) = 0 \) uniformly in \( \Phi_j(x), \psi_j(x), (j = 0, 1, ..., p - 1) \)

**Proof.** Let \( \alpha \notin Q \). By corollary 9 of [4], there exist a function \( f(\delta) \) such that

\begin{equation}
\lim_{\delta \to 0} f(\delta) = \infty, \lim_{\delta \to 0} \delta f(\delta) = 0, \quad (23)
\end{equation}

and, for any sufficiently small \( \delta \), a number \( \xi \notin Q \), satisfying (15) and (16) for all \( N \geq f(\delta) \). The same argument given in the proof of theorem 2 shows that

\begin{equation}
\|u\|^2 \leq g(f(\delta) + 1),
\end{equation}

where \( g \) is defined by (20), i.e.

\begin{equation}
\|u\|^2 \leq \frac{80\pi^2E}{(3-\sqrt{5})^2} \left[ f(\delta) \delta^p + \delta^p \right] \frac{2k}{f(\delta)} + \frac{4E}{f(\delta)^{2k}}.
\end{equation}

By (23), this yields

\begin{equation}
\lim_{\delta \to 0} (\text{Diam} \Gamma_\delta) = 0.
\end{equation}

\[ \square \]

**References**


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MATHEMATICAL MODELING

MSC 37C70

MATHEMATICAL MODELING OF THE LAW OF CLOUD DROPLET CHARGE CHANGE IN FRACTAL ENVIRONMENT

T.S. Kumykov\(^1\), R.I. Parovik\(^2, 3\)

\(^1\) Institute of Applied Mathematics and Automation, 360000, Nalchik, Shortanova st., 89a, Russian
\(^2\) Institute of Cosmophysical Researches and Radio Wave Propagation Far-Eastern Branch, Russian Academy of Sciences, 684034, Kamchatskiy Kray, Paratunka, Mirnaya st., 7, Russia
\(^3\) Vitus Bering Kamchatka State University, 683031, Petropavlovsk-Kamchatsky, Pogranichnaya st., 4, Russia

E-mail: macist20@mail.ru, romanparovik@gmail.com

The paper proposes a new mathematical model of cloud droplet charge change in storm clouds. The model takes into account the fractal properties of storm clouds, and the solution was obtained using the apparatus of fractional calculus.

Key words: fractal dimension, the mathematical model, operator Riemann-Liouville, operator Caputo

Introduction

During the last decay many geophysicists study intensively the fractality of environment structures and its effect on different geophysical processes. A cloud also refers to such natural phenomena where the question on electric charge formation and separation is a topical one. Many researches are devoted to the investigation of the regularities of electric charge separation in clouds. The main results are summarized in classical papers \^[1]-[9] where many explanations are presented not taking into account environment fractality. The results of the study in this area show that one of the important prerequisites for electric charge separation in clouds are the ice phase (ice crystals, small hail and hailstones) and supercooled water droplets \^[10].

It is known that clouds with intensive convective currents have fractal structure and a cloud is a fractal environment \^[11]. Thus, we may state that the processes occurring in such an environment are well described by the apparatus of fractional calculus.


\(^{P}arovik\ Roman Iuanoich – Ph.D. (Phys. Math.), Dean of the Faculty of Physics and Mathematics Vitus Bering Kamchatka State University, Senior Researcher of Lab. Modeling of Physical Processes, Institute of Cosmophysical Researches and Radio Wave Propagation FEB RAS.

Problem definition and solution

From Frenckel’s theory [13] in the paper [12], average charge \( q_r \) which is generated by one cloud droplet with radius \( r \) was obtained for cloud droplets in slightly ionized air environment in the form

\[
q_r = 4\pi \varepsilon_0 n \zeta a,
\]

where \( \varepsilon_0 \) is the electric constant; \( a \) is the bubble radius; \( \zeta \) is the electrokinetic potential; \( n \) is the number of bubbles with radius \( a \) formed in a cloud droplet with radius \( r \).

Thus, relying upon the Frenckel’s theory, the droplet total charge may be written in the following form:

\[
q(x,t) = 4\pi \varepsilon_0 \zeta R(x,t),
\]

where \( R(x,t) \) is the droplet radius.

The law of droplet charge change may have the form:

\[
\frac{\partial q(x,t)}{\partial t} = 4\pi \varepsilon_0 \zeta \frac{\partial R(x,t)}{\partial t}.
\]

In equation (3) the value \( j(x,t) = \frac{\partial q(x,t)}{\partial t} \) is the charge flux which depends on the velocity of droplet radius change \( \frac{R(x,t)}{\partial t} \) coinciding with the diffusive flux by the droplet surface if they grow due to the diffusion from the surrounding environment [14].

Since the process takes place in a fractal environment, than instead of the model (3) we consider the law of droplet charge change taking into account the fractality. But before the consideration of the law of droplet charge change, it is necessary to consider the droplet size change taking into account the fractality as long as charge change on the whole occurs due to the drop size change.

It is known [15] that flux equation is expressed by the formula

\[
q(x,t) = -k D^\alpha_{ax} u(x,t), \quad 0 < \alpha < 1,
\]

where \( k \) is «diffusion» coefficient; \( u(x,t) \) is the concentration (temperature and so on), \( D^\alpha_{ax} \) is the integrodifferentiating operator in the sense of Riemann-Liouville of fraction order \( \alpha \) with the initial point \( a \) which is determined as follows [16]:

\[
D^\alpha_{ax} u(\xi,t) = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial x} \int_a^x u(\xi,t) \frac{d\xi}{(x-\xi)^\alpha}.
\]

The substitution of \( \partial/\partial t \) by \( D^\alpha_{ax} \) in differential equations includes implicitly the additional factors of physical system interaction. Thus, we may state that equation (4) describes a fractal process [15].

Taking into account the relations \( j(x,t) = \frac{\partial q(x,t)}{\partial t} \) from (3) and (4) we obtain:

\[
j(x,t) = -\frac{k}{4\pi \varepsilon_0 \zeta} D^\alpha_{0t} R(x,t).
\]

Denoting by \( \lambda = -\frac{k}{4\pi \varepsilon_0 \zeta} \) and substituting the flux value \( j(x,t) \), formula (5) с учетом (3) has the form:

\[
\frac{\partial R(x,t)}{\partial t} - \lambda D^\alpha_{0t} R(x,t) = 0.
\]
Formula is the partial differential equation of the first order. We add the starting and edge values to equation (6) [11]:

\[ R(x,0) = r_1(x), x \in [0,L], \]  

\[ \lim_{x \to 0} D_{0x}^{a-1} R(x,t) = r_2(t), t \in [0,T], \]  

Solution of the problems (6)-(8) has the following form [17]:

\[ R(x,t) = \int_0^x \frac{r_1(s)}{x-s} e^{1,0}_{1,\alpha} \left( \frac{-\lambda t}{(x-s)\alpha} \right) ds + \lambda \int_0^t \frac{r_2(\eta)}{x} e^{1,0}_{1,\alpha} \left( \frac{-\lambda (t-\eta)}{x\alpha} \right) d\eta. \]  

where \( e^\nu_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \nu) \Gamma(\delta - \beta n)} \) is the Wright-type function.

Substituting (9) into formula (2), we obtain the expression for droplet charge taking into account the environment fractality.

\[ q(x,t) = 4\pi \varepsilon_0 \zeta \left[ \int_0^x \frac{r_1(s)}{x-s} e^{1,0}_{1,\alpha} \left( \frac{-\lambda t}{(x-s)\alpha} \right) ds + \lambda \int_0^t \frac{r_2(\eta)}{x} e^{1,0}_{1,\alpha} \left( \frac{-\lambda (t-\eta)}{x\alpha} \right) d\eta \right]. \]  

Considering (10) charged particle flux has the form:

\[ j(x,t) = 4\pi \varepsilon_0 \zeta \frac{d}{dt} \int_0^x \frac{r_1(s)}{x-s} e^{1,0}_{1,\alpha} \left( \frac{-\lambda t}{(x-s)\alpha} \right) ds + \lambda \frac{d}{dt} \int_0^t \frac{r_2(\eta)}{x} e^{1,0}_{1,\alpha} \left( \frac{-\lambda (t-\eta)}{x\alpha} \right) d\eta. \]  

Applying the following rule:

\[ I'(t) = \int_{x_1(t)}^{x_2(t)} f(x,t) dx + f(x_2(t),t)x_2'(t) - f(x_1(t),t)x_1'(t), \]

to (11), we obtain

\[ j(x,t) = 4\pi \varepsilon_0 \zeta \left[ \int_0^x \frac{r_1(s)}{x-s} \frac{d}{dt} e^{1,0}_{1,\alpha} \left( \frac{-\lambda t}{(x-s)\alpha} \right) ds + \lambda \frac{d}{dt} \int_0^t \frac{r_2(\eta)}{x} e^{1,0}_{1,\alpha} \left( \frac{-\lambda (t-\eta)}{x\alpha} \right) d\eta \right] = \]

\[ = 4\pi \varepsilon_0 \zeta \int_0^x \frac{r_1(s)}{x-s} \frac{d}{dt} e^{1,0}_{1,\alpha} \left( \frac{-\lambda t}{(x-s)\alpha} \right) ds + \lambda \frac{d}{dt} \int_0^t \frac{r_2(\eta)}{x} e^{1,0}_{1,\alpha}(0) + \]

\[ + \lambda \int_0^t \frac{r_2(\eta)}{x} \frac{d}{dt} e^{1,0}_{1,\alpha} \left( \frac{-\lambda (t-\eta)}{x\alpha} \right) d\eta. \]

Considering properties [17]:

\[ e^{1,0}_{1,\alpha}(0) = 1, D_{0x}^{\nu} z^{\delta-1} e^{\mu,\delta}_{\alpha,\beta}(\lambda z^\alpha) = z^{\delta-\nu-1} e^{\mu-\nu,\delta}_{\alpha,\beta}(\lambda z^\alpha) \]
Result in the final form:

\[ j(x,t) = 4\pi\varepsilon_0\zeta \left( \frac{\lambda r_2(t)}{x} + \int_0^x \frac{r_1(s)}{(x-s)^\alpha} e^{0,0} \left( -\frac{\lambda t}{(x-s)^\alpha} \right) ds \right) + \]

\[ + 4\pi\varepsilon_0\zeta\lambda \int_0^t \frac{r_2(\eta)}{x\eta} e^{0,0} \left( -\frac{\lambda (t-\eta)}{x\eta} \right) d\eta. \]  

Expression (13) is the law of cloud droplet charge change considering the environment fractality by the Wright-type function.

In the paper [11], an equation of (4) type with Caputo fractional derivative operator was obtained:

\[ q(x,t) = \gamma\partial_0^\alpha u(x,\tau), \quad 0 < \alpha < 1, \]  

where \( \gamma > 0 \), \( \partial_0^\alpha u(x,\tau) = D_0^\alpha D_\tau u(x,\tau) \) is the regularized fractional derivative of the order \( \alpha \) from function \( u(x,\tau) \) with initial and end points 0 and \( \tau \) (Caputo derivative). Taking into account formula (14) and the law of droplet size change, formula (3) is written in the form:

\[ \partial_0^\alpha R(t) - kr(t) = 0, \]  

where \( k = \frac{1}{\gamma} \). Formula (15) is an ordinary differential equation of fractional order. Add an initial condition to equation (15):

\[ R(x,0) = R_0. \]  

Since \( f(x) = 0 \), the solution of problem (16) for equation (15) has in general view the following form:

\[ R(t) = R_0 E_{\alpha,1}(kt^\alpha), \]  

where \( E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \) is the Mittag-Leffler-type function [17]. Substituting (17) into the corresponding formulas for the charge and charged droplet flux, we obtain

\[ q(t) = 4\pi\varepsilon_0\zeta R_0 E_{\alpha,1}(kt^\alpha), \quad j(t) = 4\pi\varepsilon_0\zeta R_0 t^{\alpha-1} E_{\alpha,\alpha}(kt^\alpha). \]  

Formula (18) is the law of droplet charge change in Frenkel’s generalized theory in cloud environment by means of Mittag-Leffler function.

Conclusions

Considering the clouds which are known to have different structure and have different classification in origin and morphological features to which the data on their fractal structure may be added, formation of a more general view of cloud physics state is possible in the future.

The paper suggests the mathematical model for the droplet charge change in fractal cloud environment generalizing Frenkel’s theory. The solution of this model was obtained taking into account Write- and Mittag-Leffler-type functions.
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MATHEMATICAL MODELING OF NONLOCAL OSCILLATORY DUFFING SYSTEM WITH FRACTAL FRICTION

R.I. Parovik¹, ²

¹ Institute of Cosmophysical Researches and Radio Wave Propagation Far-Eastern Branch, Russian Academy of Sciences, 684034, Kamchatskiy Kray, Paratunka, Mirnaya st., 7, Russia
² Vitus Bering Kamchatka State University, 683031, Petropavlovsk-Kamchatsky, Pogranichnaya st., 4, Russia
E-mail: romanparovik@gmail.com

The paper considers a nonlinear fractal oscillatory Duffing system with friction. The numerical analysis of this system by a finite-difference scheme was carried out. Phase portraits and system solutions were constructed depending on fractional parameters.

Key words: Gerasimov-Caputo operator, phase portrait, Duffing oscillator, finite-difference scheme

Introduction

Investigation of nonlinear oscillatory system is of great practical importance [1]. With the development of the theory for modeling of fractal processes, the possibility to determine new properties of nonlinear fractal oscillatory systems appeared. Such oscillatory processes are described by differential equations with fractional derivatives [2]. Fractional orders of derivatives are associated with fractal dimension of a medium, and consideration of them in an oscillatory system as complementary degrees of freedom give prerequisites for new chaotic regimes which describe real processes and phenomena. For example, the paper [3] investigates the question on modeling of damped oscillation in a vehicle tire. The paper [4] studies viscoelastic properties of beams, plates, and cylindrical shells.

Investigation of nonlinear oscillatory system with friction (Duffing oscillator) is of interest. The papers [5, 6] consider modeling of Duffing oscillator with fractal friction. The present paper makes a generalization of the suggested earlier models for Duffing oscillator, when instead of a displacement second-order derivative, an operator of fractional differentiation is introduced into an initial equation. Regimes of an oscillatory system in the result of change of fractional parameters are under the investigation. Phase portraits are constructed.
Problem definition

Find a solution $x(t)$, where $t \in [0, T]$, satisfying the equation

$$\frac{\partial^\alpha_0 x(x)}{\partial t^\alpha} + a\frac{\partial^\beta_0 x(x)}{\partial t^\beta} - x(t) + x^3(t) = \delta \cos(\omega t)$$  \hspace{1cm} (1)

and the initial conditions

$$x(0) = x_0, \quad \dot{x}(0) = y_0$$  \hspace{1cm} (2)

where $\frac{\partial^\alpha_0 x(x)}{\partial t^\alpha} = \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-\eta)^{\alpha-1} \frac{\partial x(\eta)}{\partial \eta} \, d\eta$, $\frac{\partial^\beta_0 x(x)}{\partial t^\beta} = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-\eta)^{-\beta} \frac{\partial x(\eta)}{\partial \eta} \, d\eta$ are operators of fractional differentiation in the sense of Gerasimov-Caputo of the order $\alpha$ and $\beta$; $\dot{x}(t) = dx/dt$, $x(t) = d^2x/2t^2$; $x_0, y_0, \delta, \omega, a, T$ are given parameters.

We should note that in the papers [2, 5, 6], differentiation operator of fractional order in the sense of Riman-Liuvill was used to describe friction. We applied Gerasimov-Kaputo operator, in this case local conditions (2) are true. In the case with Riman-Liuvill operator, it is necessary to specify nonlocal conditions [7].

Method of solution

The problem (1), (2) is solved by numerical methods, explicit finite-difference scheme. Introduce $\tau$, the sampling interval, and $t_j = j\tau, \; j = 1, 2, \ldots, N, N\tau = T$, $x(j\tau) = x_k$. Then fractional derivatives, entering equation (1), may be approximated as follows [7]:

$$\frac{\partial^\alpha_0 x(x)}{\partial t^\alpha} \approx \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} \sum_{k=0}^{j-1} (k+1)^{2-\alpha} - k^{2-\alpha} \left( x_{j-k+1} - 2x_{j-k} + x_{j-k-1} \right)$$  \hspace{1cm} (3)

$$\frac{\partial^\beta_0 x(x)}{\partial t^\beta} \approx \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \sum_{k=0}^{j-1} (k+1)^{1-\beta} - k^{1-\beta} \left( x_{j-k+1} - x_{j-k} \right).$$

Substituting the relations (3) into equation (1), we obtain the following explicit finite-difference scheme:

$$x_1 = Ax_0 - Cx_0^3 + K, \; x_2 = Ax_1 - Bx_0 - Cx_1^3 + K \cos(\omega \tau),$$

$$x_{j+1} = Ax_j - Bx_{j-1} - Cx_j^3 - B \sum_{k=1}^{j-1} b_k \left( x_{j-k+1} - 2x_{j-k} + x_{j-k-1} \right) -$$

$$- M \sum_{k=1}^{j-1} c_k \left( x_{j-k+1} - x_{j-k} \right) + K \cos(\omega j \tau)$$  \hspace{1cm} (4)

$$A = \left( \frac{2\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} + 1 \right) / \left( \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \right),$$

$$B = \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} / \left( \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \right),$$

$$K = \delta / \left( \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \right),$$

$$C = 1 / \left( \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \right),$$

$$M = \frac{\tau^{-\beta}}{\Gamma(2-\beta)} / \left( \frac{\tau^{-\alpha}}{\Gamma(3-\alpha)} + \frac{\tau^{-\beta}}{\Gamma(2-\beta)} \right).$$
\[ b_k = (k + 1)^{2-\alpha} - k^{2-\alpha}, c_k = (k + 1)^{1-\beta} - k^{1-\beta}, j = 2, \ldots, N - 1. \]

The derivative \( y(t) = \dot{x}(t) = dx/dt \) is approximated by a finite difference: \( y_j = \frac{x_j - x_{j-1}}{\tau} \).

Values \( x_0 \) and \( y_0 \) are determined from the initial conditions (2).

**Modeling results**

Numerical modeling was carried out taking into the account the following parameter values in solution of (4): \( N = 4000, \ \tau = \pi/100, \ \omega = 1, \ \delta = 0.3, a = 0.15, x_0 = 0.2, y_0 = 0.3. \)

Phase portrait is drawn according to the points \( (x(t), y(t)) \) depending on \( \alpha \) and \( \beta \) parameters.

To study the vibrational modes often use the Poincaré section. Poincaré section is a plane in the phase space, selected in such a way that all paths belonging to the attractor, crossed it under a non-zero angle.

Note, that the closed phase trajectories form a finite sequence of points in the Poincaré section (one point corresponds to the limit cycle with period \( T \), two points correspond to the limit cycle with twice the period \( 2T \), non-recurring modes correspond to the infinite sequence of points in the Poincaré section. As a cross-section Poincaré choose the plane of constant phase of external influence \( \omega t_n = 2\pi n \), which corresponds to the choice of the points of the phase trajectory exactly the period \( T = 2\pi \) external force.

**Fig. 1.** Phase portrait and point Poincaré section (a), constructed in accordance with the numerical solution of (c), taking into account parameters: \( N = 30000, \ \tau = \pi/100, \ \omega = 1, \ \delta = 0.3, a = 0.15, x_0 = -1.3311, y_0 = -0.1429, \ \alpha = 2, \ \beta = 1; \) b) this Poincaré section at \( N = 5 \cdot 10^5 \) with the same parameter values.

Fig. a case \( \alpha = 2, \ \beta = 1 \), corresponding to the classical Duffing oscillator with friction. In this case, the memory effect in the vibrating system disappears. The solution is not periodic, and it has a chaotic character (Fig. a). Confirmation of the chaotic regime for forced oscillations of fractal Duffing oscillator can be seen in Fig. b, which shows the Poincaré section, built with a large number of points of \( N = 5 \cdot 10^5 \), and the shift function \( x(t) \), which is shown in Fig. c. Based on the points of the Poincaré section
Fig. 1b, we can conclude that the classic is a bistable Duffing oscillator oscillating system [9], which has a chaotic attractor, characteristic of deterministic chaos [10].

Fig. 2 shows the phase portrait (Fig. 2a) and shift function (Fig. 2b) obtained by the numerical scheme (4) in the case of: $\alpha = 2$, $\beta = 0.6$.

![Phase portrait and point Poincaré section](image1)

Fig. 2. Phase portrait and point Poincaré section (a), constructed in accordance with the numerical solution of (b), taking into account parameters: $N = 4000$, $\tau = \frac{\pi}{100}$, $\omega = 1$, $\delta = 0.3$, $a = 0.15$, $x_0 = 1.0052$, $y_0 = 1.3901$, $\alpha = 2$, $\beta = 0.6$

It may be noted that the solution in this mode is periodic, and the phase trajectory is a limit cycle. Poincare section consists of a single point, as shown in Fig. 2b, and this point is the same as the initial point of $(x_0, y_0)$. Similar results were presented in the work [5]. You can also note that the cubic nonlinearity in the equation (1) leads to an increase in the frequency of oscillations (Fig. 2b).

Fig. 3 presents the calculated curve constructed by the formula (4). The calculation parameters: the number of points of $N = 1000$, sample rate $\tau = 0.16$, $\xi = 4$, $\alpha = 2$, $\beta = 0.8$, $(x(0), \dot{x}(0)) = (-2.623, -4.0705)$.

![Limit cycle with points of the Poincare sections](image2)

Fig. 3. The limit cycle with the points of the Poincare sections (a) and numerical two periodic solution (b) obtained by the formula (4) within the parameters: $\alpha = 2$, $\beta = 0.8$, $\tau = 0.16$, $a = 0.15$, $\delta = 4$, $(x(0), \dot{x}(0)) = (-2.623, -4.0705)$
Fig. 3 and 4. Fig. 3 b that the decision has pridelnyh cycle loop, the Poincare section contains two points. Therefore, the solution is two-periodic. Have loop leads to a bifurcation of the vibration amplitude (Fig. 3 a). Similar patterns were obtained by the authors of [5].

Fig. 4. Phase portrait and numerical solution with regard to the parameters: (a,b) $\alpha = 1.7, \beta = 1, \tau = \frac{\pi}{60}$; (c,d) $\alpha = 1.8, \beta = 1, \frac{\pi}{40}$; (e,f) $\alpha = 1.8, \beta = 0.2, \tau = \frac{\pi}{60}$; (g,h) $\alpha = 1.3, \beta = 0.2, \tau = \frac{\pi}{60}$

Fig. 4 illustrates solution evolution and phase portraits for different parameters $\alpha, \beta$ and $\tau$. In Fig. 4c, mainly phase trajectories reach the boundary cycle. In Fig. 4e, chaotic regime is observed.

It can be concluded that the emergence of new parameters (fractional exponents) in hereditarity equation widens properties Duffing oscillator and anticipates the emergence of new modes and effects in a nonlinear oscillatory systems. Orders fractional derivatives act as control parameters that define the fractal vibrational modes of the system that need to be considered when modeling.
Conclusions

The paper presents a model of fractal Duffing oscillator with friction. Numerical solutions were obtained depending on fractional parameters $\alpha$ and $\beta$. Phase trajectories were drawn. Solution analysis showed that there are both periodic solutions and chaotic regimes. For a more qualitative analysis in future, bifurcation diagrams will be drawn, and a test to determine the conditions for periodic solutions will be made.

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According to the developing model, the nonpotential part of the geomagnetic field is due to the vertical current associated with positive charge transfer by water vapour during plant and water surface evaporation in the same direction and with negative rain current in the opposite direction. These two processes are quite irregular both in space and in time, but the total charge transferred upwards to the clouds is almost equal to the charge transferred downwards to the Earth surface. Nevertheless, these processes result in the accumulation of positive charge in the lower ionosphere at the height of about 90 km.

Key words: nonpotential part of geomagnetic field, Schmidt-Bauer currents, rain currents and evaporation currents

Introduction

Possible relation between geomagnetic field nonpotentiality, Schmidt-Bauer currents (A. Schmidt and L.A. Bauer currents) (Sch-B) and atmospheric electric current (J) has been discussed by scientists for more than 100 years. However, the problem has not been solved. The reason is that geomagnetic field is considered with high accuracy to be potential, and the Sch-B currents are considered to be nonreal. Moreover, according to the estimates of field nonpotentiality value, Sch-B current intensity exceeds atmospheric current by four orders, which also seems to be unnatural.

In 1895, A. Schmidt was the first one who showed that the Earth magnetic field includes a nonpotential part (references in [Schweidler, 1936]). It is known that in the potential field, line integral along a closed curve should be equal to zero. In reality, it is not quite so in the geomagnetic field. It turns out during such an operation that zero is not always obtained. This fact indicates the existence of vertical currents reaching the...
Earth surface. Schmidt was doubtful of the reality of the estimate results, and attributed them to the inaccuracy of observations of magnetic elements [1].

Later Bauer [1] confirmed Schmidts results during recalculation of larger and better observational material. The most surprising was that the currents associated with the Earth magnetism are partially directed upwards and partially downwards to the Earth surface. The density of these currents, called Schmidt-Bauer currents, is about 10,000 times higher than that of normal atmospheric electric current. Sch-B currents evidently have another source than atmospheric current. It is quite possible that they are associated with each other. For example, one may suppose that the difference between the oppositely directed Sch-B currents provides the light ion atmospheric current and so on.

In the result of calculations carried out by Schmidt and Bauer, it was discovered that in polar regions the current is directed upwards (Fig. 1), whereas in the equatorial belt the current is oppositely directed, on the whole. Densities of the both currents are almost equal. The Earth surface, where the currents flow upwards, is almost equal to the Earth surface with the currents flowing downwards. It follows from Fig. 1 that there is no difference in current directions flowing over oceans and continents.

![Fig. 1. Schmidt-Bauer currents [1] in Northern and Southern hemispheres. In the pole regions the currents are positive (directed upwards), in the equatorial zone the currents are negative (dark regions, currents are directed downwards)](image)

**Nonpotential field**

The nonpotential part of the geomagnetic field is not only in the constant field but also in the variable field, i.e. in diurnal $S_q$ - variations [2]. In particular, Benkova showed that the nonpotential part of $S_q$ -variations reaches 1/5 of the whole variation field, and the density of variable "currents", which generate it, is two or three orders higher than the conduction current density of the atmosphere.

Serious discussion of the question about the nondipole part of the geomagnetic field $B$ has lasted for more than several decades [3]. The nonpotential magnetic field is usually ignored in the well-known textbooks, such as, «Geomagnetism» (Chapman and Bartle),
Currents in the Earth atmosphere

Currents flowing in the Earth atmosphere are known:
- Ion atmospheric current density in the regions with fair weather conditions is $2 \times 3 \times 10^{-12} \text{A/m}^2$.
- Density of the current determined by charge transfer on rain, hail, snow drops during calm showers is $10^{-7} \text{A/m}^2$.
- Density of the current determined by charge transfer on rain, hail, snow drops during thunderstorm showers and hail is up to $10^{-6} \text{A/m}^2$.
- Lightning current intensity is up to 500 kA, (with the utmost probability in the range of 20-40 kA).

Lightning voltage is up to $10^9 \text{V}$, lightning length reaches 10 km, lightning channel diameter reaches 20 sm.

We should note that Sch-B current density ($10^{-9} \text{A/m}^2$) is almost equal to the rain current density ($10^{-8} \text{A/m}^2$). As follows from Fig. 2, rain current may change its direction remaining mainly negative.

It is obvious, that there should be a current opposite to the rain current in the atmosphere. Apparently, it is the current generated in the result of charge transfer in...
the ascending warm and humid airflow. Such flows appear, as a rule, in warm forested and humid regions of continents. According to the paper [6], ascending flows, as well as rain currents have both positive and negative charges.

Some amateur radio operators proved that. They measured atmospheric currents received by their TV antennas. During clear and dry weather, the current was always positive (from antenna to ground). When there were no clouds, it was 0.10.3 nA. As cloudiness grew, fluctuations increased and sometimes reached 7 nA. During bad weather conditions, the current was always negative. During fog and drizzling rain, it was 0.21 nA and more during rain. The maximum registered value was 14 nA. Electrified fog and drizzle drops precipitated on an antenna and gave it their negative charge. If we admit, that the effective area of a TV antenna surface reached 1 m$^2$, the results of measurements of rain current are close in value.

**Charges in the atmosphere**

Compare the values of electric charges in the Earth atmosphere:
- Average charge of a thunderstorm cloud is 50 coulombs.
- Cyclone charge according to our estimates reaches $Q = 5 \cdot 10^3$ C. Its area is $100 \times 100$ km$^2$ [7].
- The Earth charge as a ball with radius $R$ having the field $E = 100 V/m$ is $Q_1 = \varepsilon_o R^2 E = 5.7 \cdot 10^5$ coulombs, $R_E$ is the Earth radius, $\varepsilon_o$ is the electric constant.
- The charge of a 1 km positive ion layer at the height of 85 km is $Q_2 = NShe = 10^4 cm^{-3} \times 5 \cdot 10^{18}$ cm$^2 \times 1.6 \cdot 10^{-19} C = 10^9$ C, where $N$ is charge concentration, $S$ is the Earth surface area, $h$ is the layer thickness, $e$ is the electron charge [8].
- The charge transferred to the Earth by rain current during a day on the area of about 1% from the Earth total surface is $Q_3 = j \times S \times k \times t = 10^{-10} A/cm^2 \times 5 \cdot 10^{18}$ cm$^2 \times 10^{-3} \times 1$ day ($\approx 10^5$ s) $= 5 \cdot 10^{10}$ C. Here, $j$ is the rain current density, $k$ is the Earth surface fraction covered by rain, $t$ is the time for the Earth charging by rain.
- The charge of «Earth-ionosphere» capacitor $Q_4 = CU$, where $C$ is the Earth-ionosphere (electrosphere) capacitor value $C = 4\pi \varepsilon_o \Delta R_E / R_E^2 = 5 \cdot 10^{-2}$ F. $\Delta R_E$ is the ionosphere
height. $U$ is the ionosphere potential $U = 300000 \ Q_4 = CU = 5 \cdot 10^{-2} \ F \times 3 \cdot 10^5 \ V = 1.5 \cdot 10^4 \ C$.

Comparison of charge $Q_1$ and $Q_4$ values, directly relating to AEF, with the charge values of positive ion layer (hydronium ions, $\text{H}_3\text{O}^+$) $Q_2$ and the rain cloud charges $Q_3$ shows that electric charges of the atmosphere are significantly (by more than four orders) higher than those attributed to the AEF. The nature of AEF currents is known, it is light ion drift, but we cannot say the same about the nature of Sch-B currents. We have mentioned above that Sch-B currents are close to rain currents by value. Assume that the rain current is the Sch-B current. Then, the current with an opposite direction is the current of charges rising into the atmosphere by ascending air. Rain drops are negatively charged, as a rule. They fall downwards that means that the current is directed upwards. The rise of negative drops by ascending flows determines the downward current. The change of the drop charge sign result in the change of Sch-B current direction.

It is known that during evaporation of water, electric charge separation takes place in the gravitational field. This process is responsible for electric flashes and lightning in the atmosphere. Coulomb dynamic attractors focusing the electric field and the charge are formed during the dynamics of such processes. Formation of a double layer on a phase boundary is the common property (not only water and vapor). Intensive evaporation, ionization and charge separation occurs on the surface of warm, humid and forested continents. In tundra and deserts, an opposite process, precipitation, takes place. The hydrologic cycle determines electric charge cycle, and charge transfers are the Sch-B currents.

At the same time, water dissociation into hydrogen and hydroxyl ions occurs. A free ion $\text{H}^+$ is not ectogenous and is hydrated by a water molecular to form hydroxonium ion, $\text{H}^+ + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+$. An overall reaction is the transfer of a proton from one molecular to another one and production of hydroxonium and hydroxyle ions, $\text{H}_2\text{O} + \text{H}_2\text{O} \rightarrow \text{H}_3^+ + \text{OH}^-$. Hydroxonium is a volatile gas which ascends quickly in the atmosphere to the height of 85 km and gathers there for unclear reason [8]. Hydroxyl coagulates water vapor, turns to water aerosol which is raised by ascending warmed airflow. Precisely this process is the Sch-B current directed downwards. It is known that clouds are mainly formed on the equator. They are transferred by winds to colder regions of the Earth and rain down.

Observations of the change of AEF polarity carried out in Kamchatka, in the region of thermal fields of Mutnovskii volcano, confirm the idea. We showed that when steam with large content of condensed water is ejected from a thermal well, the AEF value decreases and may change its polarity. If dry water steam is ejected form a well, the AEF value significantly increases [9].

**Electric energy of the atmosphere**

Compare the electric energy accumulated in the «Earth-ionosphere» capacitor with the atmosphere energy. The electric energy is $W_C \approx Q$.

- The energy of atmospheric electric field is $W_1 = 3 \cdot 10^4 \text{ J}$, the energy of ion layer is $W_2 = 5 \cdot 10^7 \text{ J}$, the energy of rain charge is $W_3 = 2 \cdot 10^9 \text{ J}$. The capacitor energy is $W_4 = 5 \cdot 10^2 \text{ J}$.

Compare the obtained values with the atmosphere energy. According to [10], the inner energy of the whole atmosphere is estimated to be $8.6 \cdot 10^{23} \text{ J}$, the potential one
is $3.6 \cdot 10^{23}$ J, and the kinetic one is two orders less, $10^{21}$ J, which means that it is less than 1% from the potential energy. It is clear that the electric energy of the atmosphere is negligible in comparison to the kinetic energy.

The estimates given above show that the change of Sch-B currents should be observed on the Earth depending on climate variations under the general condition of equality of upward and downward currents. Currents should change during winter-summer seasonal periodicity, and during temperature change. Evidently, investigating the data changes of observatories for the last half a century, the tendency of Sch-B current change caused by global warming trend may be discovered (Fig. 3).

Fig. 3. Schmidt-Bauer currents (on the left). Climate of Europe (on the right)

Climate and Sch-B currents

This idea was checked by the authors [11]. They discovered the phenomenon of geomagnetic field nonpotentiality in Europe. The authors showed that induction rotor may be distinguished in the variable geomagnetic field. The negative sign of the induction rotor corresponds to negative current, and this current is directed from the atmosphere to the Earth surface. The positive sign of the induction rotor corresponds to positive current. Current density may reach $\pm 1 \mu$ A/m$^2$. The preliminary results obtained by the authors [11] show that in northern and southern regions of Europe, negative currents (directed to the Earth surface) flow. Positive currents (directed to the atmosphere) flow in the central part of Europe (Fig. 3 on the left).

If we refer to the schematic view of European climate (Fig. 3 on the right).

We may see (Fig. 4) that there are no necessary conditions (forestlands) for the formation of ascending humid flows transferring negatively charged water aerosols into the atmosphere in the northern (Tundra) and southern (Semiard) regions. Most likely, precipitation and transfer of rain negative electricity to the Earth takes place in these regions. The largest upward flows are formed in the forest regions where evaporation is the most intensive. During evaporation, ionization and formation of negatively charged aerosols occur, since it is the water property to keep negative charge.

Positive ions of mainly hydroxonium are raised by ascending flows, and, finally, gather at the height of about 80 km. They do not participate in Sch-B currents.
The result obtained by the authors [11] does not contradict our model of Sch-B currents. However, it is difficult to compare it with the known pattern of Sch-B currents illustrated in Fig. 1. This pattern was obtained about 100 years ago and has not been updated.

Ideas, described in this paper, require proving the reality of existence of geomagnetic field nonpotentiality and Sch-B currents. There are all the grounds to realize such investigations (data bank of geomagnetic observatories).

It also makes sense to repeat the study of diurnal $S_q$ - variations made by N.P. Benkova [2]. Such investigation was performed by V.V. Plotkin [12]. The author carried out spatial interpolation of time harmonic complex amplitudes of field component observations obtained at sites and represented according to the universal time. Complex annual average amplitudes of diurnal $S_q$ - variation time harmonics obtained according to the data of 132 stations of the global network for 1958 were used.

The author succeeded to show that there is a field nonpotential part in $S_q$-variations besides the potential one. It is comparable with the potential part and is about 10 nTl. If we take the atmospheric current into the account, the vortex part of its magnetic field should be 0.01 nTl. The author ascribes the nonpotential part to local noise of accidental origin at the sites. If we assume that the nonpotential part of the geomagnetic field discovered by Plotkin [12] is due to some other vertical current, its density may be of the order $10^{-9}$ A/m$^2$. It is quite possible that this current is the Sch-B current or the rain current according to our model.

**Conclusion**

According to the suggested model, Sch-B current have climatic roots. This approach did not quite agree with the results illustrated in Fig. 1, as we wished it to be. Confirmation of the results obtained by Schmidt and Bauer on the basis of modern
data should show the validity or falsity of our model. Obviously, in the second case, we will have to find another ways to solve the problem of nondipole geomagnetic field.

To realize large-scale observations of Sch-B currents, simple devices for registration of rain currents and ascending flow currents should, possibly, be constructed and installed at different stations and observatories.

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The proposed model presents a rock bump (a technogeneous earthquake) as a shock wave emergence on the inside wall of a mine. In such a case, an unloading, tensile wave is generated which moves in the opposite (into the wall) direction. A shock wave is accompanied by medium motion in the direction of shock wave propagation with the velocity significantly less than that of the wave. Phenomena, occurring on the inner surface of a mine result in coal dustfall and methane dissolution in the coal that causes an explosion.

Key words: nonpotential part of geomagnetic field, Schmidt-Bauer currents, rain currents and evaporation currents

Introduction

Within the developed shock-wave model of an earthquake, the author considers a rock bump as a technogeneous earthquake. Possible consequences of a rock bump such as fast methane and coal dust emissions followed by component outbursts are discussed. These phenomena occur in the result of the emergence of a shock wave at the surface of a coal bed. A shock wave is generated in a rock as the medium reaction on lithostatic unbalance due to the disturbance of its integrity during shaft work, i.e. due to the formation of rock pressure.

At one of the largest coal mines in the world, at «Raspadskaya» mine in the South of Kuzbass, explosions with the interval of four hours occurred at night on May 9, 2010. The second explosion was significantly stronger than the first one and it occurred at the time when rescuers had already descended into the mine. People died. The cause for the explosion is considered to be methane and coal dust emissions and careless handling of fire. May a rock bump (technogeneous earthquake) be the cause of the tragedy? To answer this question, we shall give some information on a rock bump.

About 3 thousand rock bumps (RB) are registered annually at the North-Ural deposit of bauxite. There is a Monitoring and Warning Service for RBs, local underground
Earthquakes with rock displacements, at the North-Ural Bauxite Mine (SUBR). The strong rock bump which occurred at this mine on February 13, 2010 was identified as an earthquake by seismologists (http://echo.msk.ru/news/656402-echo.phtml). The Geophysical Service RAS gave the magnitude of 4.7 to this technogeneous event. The Geophysical Service of USA gave the magnitude of 4.6. A question arises, may the magnitude (M) of a rock bump be more than, for example, 6 or 7, if not, what maximum magnitude may a RB have?

Model

Earthquake sources, ranged according to the magnitude, are orderly distributed not only in time (Guttenberg and Richter law) but in space as well [1]. It turned out that average statistical distances \( d_M \) (km) between the epicenters of the closest pairs of seismic sources with the size \( L_M \) (km) and the magnitude are described by the relations:

\[
    d_M = 10^{0.6M-1.94}, \quad L_M = 10^{0.6M-2.5}.
\]

The value \( d_M \) practically characterized the average size of geoblocks capable to generate earthquakes with magnitude. The coefficient 0.6 by indicates the change of source size \( L_M \) and the corresponding distances between epicenters \( d_M \) by 2 times for each successive step of 0.5 of a magnitude unit. The value \( d_M/L_M \) is equal to 3.63 and does not depend on earthquake magnitude. For example, for the magnitude \( M = 3 \), \( d_M \approx 700 \text{ m} \), and \( L_M \approx 200 \) (Fig. 1).

![Fig. 1. Earthquake magnitude M, geoblock size d_M, fracture length L_M [1]. Thin lines are the data on fracture length of different authors. On the insert: dependence of M - δ. Dots show earthquake parameters in Sumatra (i), Altai (ii) and California (iii).](image)

Numerous observations of seismologists allowed us to detect the relation between earthquake magnitude and fracture length (thin lines in Fig. 1). For a RB, the fracture...
length is associated with mine size (a geoblock). According to [1], the magnitude \( M = 4.7 \) corresponds to \( L \approx 2.5 \) km, and \( d_M \approx 8 \) km. Linear size of a technogenuous earthquake source could reach such a value. Estimate the maximum possible values of \( L \) and \( d_M \) taking into account the known size of this mine. The Raspanskaya mine field size: strike length is 12.7 km, vertical extent is 4.3 km, area is 54.5 sq.km. (http://premier.gov.ru/-visits/ru/11167/info/11170/). It is natural to consider the maximum mine size to be the geoblock size. According to Fig. ??, when a geoblock has the size of \( d_M \approx 13 \) km, the maximum magnitude of an earthquake may reach \( M = 5 \), the fracture length is \( L_M = 3.8 \) km. It is a very strong earthquake. We would like to remind that these are the maximum possible values for a mine with Raspadskaya mine size. For example, for an earthquake with the magnitude of \( M = 6 \), \( L_M \approx 15 \) km, and \( d_M \approx 50 \)–60 km, which is evidently larger than the medium size of the mine. In this case, we deal with a tectonic earthquake. Here is one more example. The source size of one of the strongest earthquakes occurred on December 26, 2004 near Sumatra island with \( M = 9 \) is \( L_M \approx 1000 \) km.

For more than a hundred of years, the mankind has been trying to answer the question, what an earthquake is. As it is known, there is no a simple answer for the question. According to the version of experts and readers of the popular LiveScience journal, one of the ten mysteries of the Universe is formulated as follows: What occurs in the hart of an earthquake? It is not a casual question. Modern seismology cannot answer it, nevertheless, we shall try to answer it realizing that nobody knows the definite answer so far, and any attempt to do it may be only some approaching to it.

The author suggests principally new physics of an earthquake, the so called shock-wave (SW) model [2, 3]. According to this model earthquake phenomenon may be presented as three subsequent stages of one process: 1 – SW initiation deeply inside the Earth; 2 – SW propagation from a hypocenter to the Earth surface and 3 – SW emergence at the Earth surface. During the SW emergence at the surface, an unloading (rarefaction) wave is generated which interacting with the original SW causes generation of so called “strong motions” on the Earth surface, such as: ground fractures and splits, loosening, rise and fall of water level, ground vibration similar to that of a liquid, spring formation and so on. All these processes are characteristic to the phenomena occurring during SW emergence at the surface of a solid [4]. The peculiarity of a SW is the fact that, in contrast to acoustic waves, behind the SW front, mass transfer takes place with the velocity significantly less than that of a SW. Mass transfer is a well characterized fact, which is called a slip in seismology. It is usually interpreted as a frictional sliding of a material along a formed fracture and it is a consequence of a SW propagation through a solid. In the generally accepted model of modern seismology, this phenomenon is called “emergence at the surface of an earthquake source”. In principle, such interpretation is applicable in our model, but instead of a fracture emergence (which is not real) we understand the processes accompanying a SW emergence at the surface.

Let’s consider a model, where a weak shock wave is involved. Such a wave, for example, cannot melt the substance of the medium where it propagates and, moreover, evaporate it [4]. The finite state of a substance after the unloading is supposed to be solid. The final volume of the unloaded substance \( V_x \) differs little from the normal volume of a solid \( V_o \). At the same time, we shall consider the shock wave to be not too weak to neglect the effects associated with solid strength. Pressure in a body compressed by a shock wave is supposed to be isotropic just as in gas or in liquid. It is fair when the pressure is great in comparison to ultimate strength, critical shear stress and so on.
In this case, the sound velocity is determined by the compressibility of a substance, compression modulus just as in gas or in liquid.

Let a plane shock wave with constant amplitude (pressure $p$, mass velocity $u$, volume $V$, which is a little less than the normal volume $V_o$) propagate through a solid. At the definite time, the wave emerges at a free surface which is considered to be parallel to the shock wave front surface. A weak shock wave, where the compression is small, $V_o - V << V_o$, does not differ from the acoustic compression wave and is described by acoustics formulas. It propagates through the body with sound velocity $c_o$. The pressure in it is related with mass velocity as $p = \rho_o c_o u$ ($\rho_o = 1/V_o$). From the moment $t = 0$ of the shock wave emergence at a free surface, an unloading wave, which is also an acoustic one, propagates backward though the body. In a substance it has sound velocity (which slightly differs from sound velocity in normal conditions $c_o$). Wave pressure drops from the initial to zero, and the substance acquire the velocity $u'$, associated with pressure change $\Delta p = -p$ by the acoustic formula: $u' = -\frac{\Delta p}{\rho_0 c_0} = \frac{p}{\rho_0 c_0}$ (Fig. 2); density slightly decreases: final density $\rho'$ differs a little from normal density of a solid, $(V_1 - V_o << V_o)$. It is clear from the comparison of the formulas $p = \rho_o c_o u$ and $u' = p/\rho_o c_o$ that the additional velocity acquired by a substance during the unloading $u'$ is equal to the mass velocity in a shock wave, i.e. when a weak shock wave emerges at a free surface, substance velocity doubles, $u_1 = u + u' \approx 2u$. Let’s see, what effects a SW emergence at the inner surface in a mine may have.

![Fig. 2. Profiles of density, pressure and velocity during a weak shock wave emergence at a free surface a) before the emergence $t < 0$; b) after the emergence $t > 0$](image)

The section marked by gray shows that media loosening occurred after a SW passage, and a layer with the thickness $\delta$ and the density $\rho_1 < \rho_o$ was formed.

It is known that in the mines at the depth of 300-600 m, the so called dynamic effects of rock pressure in the form of rock «bursts», shocks and rock bumps are sometimes observed. «Burst» occurs as a rebound of a rock piece from the stressed massive accompanied by sharp sound. A shock or a rock bump of the internal action is a failure deep in rocks without an outburst into the mine. Its external manifestations are sharp sound, rock shaking, rock falling from the mine surface, and an air wave during strong shocks. It follows from the aforesaid that these phenomena accompany a shock wave emergence at a «free» surface.
As an example, we shall show the effects of «bursts» of tank inner armor fragmentations formed after a composite shell impact on the outer side. These effects are well known and studied. Bursts of armor fragmentations and injuries of tank crew occur in the result of emergence of a shock wave formed after a shell impact on the tank armor, its inner surface. Similar effects are observed at the moment of earthquakes. “Bursts” usually occur in rocky ground. When there are sedimentary water-saturated rocks, springs, water fountains, are formed as it occurred, for example, during the Chuiskoe earthquake, Altai, in 2004 [3].

Let’s estimate the value of substance loosening $\delta$, formed after a SW emergence. It is likely to be proportional to earthquake magnitude, so suppose that $\delta$. According to the observations during the Chuiskoe earthquake in 2003, $M = 7.5$, $\delta = 100$ cm, during the Northridge earthquake, California, in 1994, $M = 6.7$, $\delta = 50$ cm [2]. For comparison, during the earthquake in Sumatra (2004) $M = 9$, the loosening was 20 m. We construct a dependence of $\delta$ for three given earthquakes (an insert in Fig. ??). Continuing it to the region of smaller magnitude we obtain quite an approximate estimate of the loosening value for a mine with a SW $M = 5$, $\delta \approx 3$ cm. To estimate the effect, we suppose that the medium density after a SW propagation $\rho'$ is about 0.9 from $\rho$ ($\rho$ is set to be equal 2 g/cm³), then $\Delta p = 0.2$ g/cm³. We calculate the quantity of the substance emitted from the mine surface of 1 cm², or from the volume equal to 1 cm² $\times \delta = 3$ cm³. It is about 0.6 – 0.8 g. The content of the emitted substance is methane and coal dust. The relation between these two components is known, there are 30 m³ of methane for a ton of coal. In our case, there is $\approx 1$ g of coal (coal dust) and $\approx 30$ cm³ of methane. As it is known, explosive methane concentration is 5%, that means that in the air volume 600 cm³, an explosion may occur. As we have related our estimates to the coal surface area of 1 cm, the dangerous methane concentration may appear up to the distance of 6 m from the surface, on which a shock wave emerges not considering the methane diffusion rate in the mine air.

The presence of methane significantly affects the coal dust explosion. When methane is absent, the coal dust explodes if its content in the air is not less than 30-40 g/m³. When there is 2% of CH₄, the dangerous concentration of dust decreases to 10 g/m³, and if there is 3% of methane, this concentration decreases to 5 g/m³. To prevent an explosion, it is enough to decrease the dust concentration to 5 g/m³, and to 1 g/m³ allowing for some margin. Thus, the dangerous concentration of dust is considered to be 10 g in 10⁶ cm³, or for the area of 1 cm², the volume of 10⁵ cm³ should contain the volume of 1 g of dust.

Not knowing the character and the rate of dust mixing in a mine, it is not possible to calculate the time for formation of explosive concentration. Nevertheless, it is possible to estimate the range of velocities of continuum motion at the moment of emergence of a SW at the surface. We apply the data on velocities of medium motion at the moment of Northridge earthquake (1994) [2]. The uniqueness of the earthquake is that the principle shock fell within a special test field to control the so called «strong motions». In the epicenter of this earthquake, the registered velocity of medium motion was $\approx 100$ cm/s. The magnitude is higher than that accepted for estimates $M = 5$ by two units. Consequently, the velocity $u'$ should be about 100 times less, and it may be considered to be equal to 1 cm/s. For the time between two explosions, equal to about 10⁴ s, the particle will pass the distance of $l \approx 100$ meters. The linear size (distance to the wall) of the explosion volume will be not less than 100 meters. We should only find the source of explosion initiation.
It is known from the solid state physics, that during distraction (grinding) of a substance having a crystal structure and low electroconductivity, the newly formed particles acquire the electric charges of both polarities. The total charge of particles is equal to zero since the basic material is electrically neutral. All the coal put into a mill barrel after grinding is divided into two parts equal in mass with similar electric charge values but opposite in electricity sign. When the troboelectric charges are absent, particle charges acquired in the result of grinding do not manifest themselves. There is no a total electric field of any particle assembly. If the charge sign depend on particle size, which occurs with ash particles during a volcano eruption, charge separation is possible in a moving coal dust stream. For example, in an ash cloud erupted by a volcano, small particles acquire the negative charge and heavier particles acquire the positive charge [5]. Evidently, heavy ones have larger moment and propagate further than the negative ones for the «allotted time». As it follows from [5], the formed tension reaches a breakdown. Generation of electric discharges in volcano ash [6], in the dust during massive explosions and in flour production is well known. This situation is possible when the mixture of coal dust and methane or pure methane is emitted during a rock bump. A spark causes explosion of such a mixture. In the model, we considered two subsequent explosions. At first, methane explodes, and when the dust has taken quite a large volume of a mine, coal dust explodes. Another variant is possible, the first explosion is the foreshock, and the second one is the primary shock. It does not principally change the case.

The shock-wave model of an earthquake is based on new approaches in the explanation of the known experimental results obtained in the investigations of rock samples during their compression at heavy press. The authors of a number of papers observed the phenomenon of spontaneous increase of acoustic emission (AE) intensity which also spontaneously stopped (Fig. 3a) [7].

![Fig. 3. a) Fracture formation rate in a diabase when a sample is constantly influenced by monoaxial compressive stress [7]. b) Geoacoustic signals registered before the earthquake on 18.12.2002 (K = 12.1) in Kamchatka (IKIR). The event time is indicated by an arrow [11].](image)

The author [8] made an attempt to explain this phenomenon from the point of view of self-organization of a coherent structure on the basis of interaction of sound waves with opening fractures. A supposition was made that the effect of AE intensification has something in common with optical superfluorescence [9]. The interesting fact, investigated in detail in some works in Japan and China, is that the acoustic superfluorescence effect occur not in all types of rocks [2]. For example, the research of AE
regimes of granites (granodiorites) of two different types, Oshima (fine-grained) and Inada (coarse-grained), showed that the samples almost do not differ in appearance, and under the loading on a press, they behave in a differently \[10\]. On the series of granite samples for Oshima, there is a constant effect of sharp increase of AE intensity, called the acoustic superfluorescence, but there is no such an effect on the series of granite samples of Inada type. The result has been multiply confirmed. This gives ground to suppose that deep in the ground, there should be geological bodies which rheology allows self-organizing processes to be developed and, finally, earthquakes to be generated. In other bodies, which are identical to the first ones from the first sight, such phenomena cannot occur.

The results of numerous laboratory experiments and natural observations (Fig. 3a) show that at the background of constant acoustic signal \((I = dN/dt \sim N)\) emitted by a rock loaded sample, explosive growth (chain reaction type) of the number of opening fractures \(N\) (acoustic pulses) per a time unit \(t\), \(dN/dt \sim N^2\) \[7\] takes place. There is no clear understanding of the physics of this phenomenon.

Suppose that in a medium volume under an ambient pressure, some acoustic pulse sources are formed. These may be opening fractures, forming dislocations, formation and break-ups of medium hydrogen bounds and so on. Under the certain circumstances, each of these dislocations emits an acoustic pulse. The sum of such pulses is the acoustic background. Let’s imagine a situation when such dislocation is connected with other dislocations by an unclear relation so that it (the dislocation) stimulates the others to emit acoustic pulses. For example, acoustic waves, generated during the distraction of \(N\) dislocations, may promote the distraction of other \((N - 1)\) dislocations connected (linked, involved) with them. In the result, the acoustic background increases from \(N\) pulses per a time unit to \(N + N(N - 1) = N^2\). Similar processes are generally called cooperative ones in physics. The principle moment in this model is still the unclear physics of such connection. Indeed, when one tries to explain the acoustic superfluorescence of a small rock sample, the mechanism, in which an opening fracture generates an acoustic pulse which causes the opening of other prepared microcracks by a not quite clear way and everything repeats all over again, is logical. There is no limit caused by the finite sound velocity in a sample. But such mechanism is not applicable when we are talking about large distances comparable with the size of an earthquake source.

As an example, we show a record of acoustic signals, registered in Kamchatka and having the direct relation with a preparing earthquake. High-frequency acoustic signals are generated in the intermediate vicinity (not more than one or two kilometers) from a receiver. The earthquake occurs at the depth of about 30 km and at the distance of about 100 km from a receiver. Nevertheless, the receiver somehow «feels» the earthquake. Some time before the event, the acoustic background increases sharply and decreases back sharply as well. This follows by a time interval called «seismic quiescence» by seismologists (Fig. 3b) \[11\].

Is generation of a SW possible in a row? Apparently, it is, since observations of anthracite piece behavior under compression on a press shows the same picture of acoustic superfluorescence (Fig. 4) \[12\] as in the diabase in Fig. 3a.

The data, showing the evident relation between an earthquake and methane outburst are known, for example \[13\]. But there are no enough grounds to say that every explosion on a mine is due to an earthquake (rock bump). The matter is that in the case of Raspadskaya mine, the Geophysical Service SB RAS, having an extensive network of
seismical stations in the region of the explosion, could not estimate the magnitudes of the first and the second explosions due to very unclear seismogram records.

Fig. 4. Change of AE activity in time during monoaxial coal deformation in the second cycle (II) after a triaxial axisymmetric compression in the first cycle (I) [12]

Conclusions

May a rock bump be predicted? This question can be rephrased. May an earthquake be predicted, its location, time and intensity? So far, we can answer these questions only negatively. Are there any premises to solve it? Looks like, it is so. If a seismically hazardous region behaves the way it is shown in Fig. 3b, i.e, after acoustic emission intensification comes the period of seismic silence (calm), than there is a shock at the end. If we succeed to understand the physics of this phenomenon, it seems there are premises for it, than a short-term forecast of a rock bump in a mine is possible. If we succeed to find out if the duration of acoustic silence is associated with earthquake magnitude, than we shall have the possibility to predict the intensity of a rock bump. Of course, a noisy mine is not the best place for acoustic sensors. The possible solution of the problem is the significant difference in frequency ranges for the sources, natural and technogeneous ones.

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**HIGH-FREQUENCY ACOUSTIC EMISSION EFFECT**

Yu. V. Marapulets

Institute of Cosmophysical Researches and Radio Wave Propagation Far-Eastern Branch, Russian Academy of Sciences, 684034, Kamchatskiy Kray, Paratunka, Mirnaya st., 7, Russia

E-mail: marpl@ikir.ru

Complex monitoring of acoustic emission (AE) in the sound frequency range has been carried out in the Kamchatka peninsular since 1999. In the course of the investigation, the existence of acoustic emission effect in sedimentary rocks was detected. It consists in the increase of geoacoustic radiation intensity in the frequency range from hundreds of hertz to the first tens of kilohertz during the growth of rock mass deformation rate. This effect was stably observed at several spaced stations and appears the most vividly at the final stage of earthquake preparation.

During the acoustic emission effect, clear anisotropy of geoacoustic radiation directivity occurs which is determined by the source orientation of acoustic oscillations in the stress field of near surface sedimentary rocks.

*Key words: acoustic emission, acoustic activity, acoustic emission directivity, deformation rate, high-frequency acoustic emission effect*

Introduction

Acoustic emission (AE) is elastic wave emission occurring during local dynamic restructuring of solid body inner structures. The main sources of AE are plastic deformation processes associated with formation, movement and disappearance of crystal lattice defects, formation and development of micro and macro cracks as well as friction, in particular, of crack edges against each other. Acoustic emission phenomenon is observed in a wide scale range and the corresponding wave lengths of emitted oscillations. We may distinguish three frequency ranges of the emission, investigations of which differ both by tasks and by observation means. The infrasound frequency range (fractions – units of hertz), the seismic one, is used to register earthquakes and to estimate their characteristics, to monitor nuclear tests, in seismic exploration. The ultrasound frequency range from 20–30 kHz to the first MHz is used in industry for early crack recognition, detection of hidden defects in constructions of different types, as well as in geophysics during laboratory deformation of rock samples to investigate crack formation.

Marapulets Yuri Valentinovich – Ph.D. (Tech.), Associate Professor, Deputy Director for Science Institute of Space Physics Research and Radio Wave Propagation FEB RAS, Professor of Dep. computer science Vitus Bering Kamchatka State University.

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mechanisms. The sound range takes the intermediate position and plays an important role in the interaction of micro and macro dislocations, thus, acoustic emission at these frequencies is of special interest during the investigation of plastic processes in natural environments. Stability of landscapes, mountain slopes, glaciers, snow mantles and large technical constructions is associated with them. They also play an important role in the formation of earthquake precursors of different nature. The peculiarities of generation and propagation of sound range signals in complicated natural conditions retarded the development of acoustic methods for diagnostics until recently.

To register acoustic emission signals in the sound frequency range (geophysicists often use the term seismoacoustic emission), high-frequency seismic stations with magnetoelastic [1] or piezoceramic [2] seismoacoustic receivers (hydrophones) are applied. The upper limit of the operational range of such devices does not usually exceed a hundred of hertz and only in some models reaches 1 kHz. Up to the present time, the sound range of more than 1 kHz has been considered to be ineffective due to the strong attenuation of elastic oscillations at such frequencies in heterogeneous rocks [3]. The results of AE investigations carried out at the beginning of the 21st century in seismically active regions of our country, in Sakhalin [4] and Kamchatka [5], and abroad in Italy [6] showed that in the frequency range of more than 1 kHz, quite strong geoacoustic signals, including those associated with earthquake preparation, are registered. It is appropriate to call this range a high-frequency one relatively the standard range for registration in seismoacoustics. Hence, we shall hereafter use the term high-frequency acoustic emission to describe the geosignals registered in the frequency range from hundreds of hertz to the first tens of kilohertz.

Method for registration of acoustic emission in the sound frequency range

During the investigations in Kamchatka [5], it was shown that a typical signal of acoustic emission is composed of a sequence of relaxation pulses of different amplitude and duration with shock excitation and filling frequency from hundreds of hertz to the tens of kilohertz [7]. To study such signals, it was necessary to develop a hardware-software complex allowing us to register and to make analysis of acoustic emission in a wide range of sound frequencies from units of hertz to the first tens of kilohertz. Besides data registration, it was necessary to provide the possibility for data storage, analysis in different frequency ranges, ideal time synchronization as well as registration and consideration of meteorological values. Owing to the perspective of allocation of the registration system in remote hard-to-reach regions, to decrease the industrial noise effect, we had to realize remote control of the equipment organizing computer radio connection channel via a retransmitter. Fig. 1 illustrates the structure of hardware-software complex for registration and analysis of acoustic emission signals.

Owing to the absence of hydrophones capable to register geosignals within the whole range of sound frequencies and considering the results obtained during registration of seismoacoustic signals by hydroacoustic stations installed on an ocean shelve [4], piezoceramic hydrophones placed by the bottom of natural and artificial reservoirs were used as acoustic emission sensors. Results of experimental studies of signal propagation in closed inner reservoirs [5] and on an ocean shelve [8] show that at small distances, pulse-shape distortion is not significant during the propagation in a waveguide composed of a water layer and a near-surface soil layer.
Thus, investigation of hydroacoustic signals by hydrophones installed in water by the bottom of reservoirs is quite acceptable. We should note that there are no transverse oscillations in fluids. This may be applied for selection of sound waves propagating in solid mediums.

Fig. 2 shows an exemplary scheme of an acoustic experiment. Emission generation takes place in near-surface sedimentary rocks and signal registration is carried out in a fluid medium by the bottom of a reservoir. At the boundary of two mediums, refraction occurs. During the transition of longitudinal oscillations from sedimentary rocks into water, the refraction index is about 1.2 – 1.7. Taking into the account small distances of signal propagation, we may neglect the refraction effects.
In the experiments for registration of acoustic emission signals in the sound frequency range, we used a system of four piezoceramic hydrophones directed according to the cardinal points with direction diagram of 60° developed at IKIR FEB RAS.

The problem of detection of wave arrival direction was solved by vector-phase methods [9, 10]. A combined receiver (CR), constructed by ZAO «Geoakustika» at FGUP VNIIFTRI, was applied. It measures acoustic pressure and three mutually orthogonal components of pressure gradient. During the processing of these four signals, vectors of oscillating speed, shift and power density of acoustic emission are found. To detect and to determine the direction to the radiation source and to analyze geoacoustic pulse flux, an automation method was developed [7]. The method considers signal envelope form and determines its beginning. The amplitude is found from the maximum values of the envelope, then pulses and their directivity are determined. We exclude signals with increased noises from the analysis. They are revealed by the estimation of the relation of circumellipse minor and major semiaxes.

In the investigations of acoustic radiation directionality, the notions of integral $\Omega(t)$ and differential $D(\alpha, t)$ acoustic activities were used [7, 11]. The first one from these values is the pulse repetition frequency dependent on time, and the second one is the direction distribution of these pulse repetition frequencies.

A hardware-software complex was developed. It allows us to register and to determine the direction of arrival of original signal in the sound frequency range. “Wave” format of sound data was used for data storage [12]. Simultaneously with that, digital filtration in seven frequency subranges was carried out: less than 10, 30 – 60, 70 – 200, 200 – 600, 600 – 2000, 2000 – 6500, more than 6500 Hz, followed by detecting and signal collection on a 4-second interval for each subrange. To reveal the reason of anomalies in acoustic signals, their correlations with deformation and meteorological parameter measurements as well as with seismic data are under analysis.

Registration systems for acoustic emission were installed in reservoirs at three sites of complex geophysical observations of IKIR FEB RAS in Kamchatka: at “Paratunka” basic observatory (since 2008) and at remote “Karymshina” (since 1999) and “Mikizha” (since 2001) stations located at the distances of 20 km and 4 km respectively [12].

Features of high-frequency acoustic emission effect

In the course of the study it was ascertained that acoustic signals of deformation nature may be divided into pulses during background period and during the increase of rock deformation rate. Intensification of plastic process may be associated with rock loosening at the observation point or with the formation of a stress remote source [13]. During the background period, insignificant in amplitude acoustic pulses with the repetition frequency within 0.1 – 0.5 pulses per second are observed. As an example, Fig. 3a shows a 10-minute fragment of the record of such a signal. Fig. 3b illustrates an example of its energy spectrum obtained by averaging of 16 realizations of fast Fourier transform (FFT) estimated by 2048 signal samples. Thus, to construct the energy spectrum, realization of a signal 0.76 s long was used for the sampling frequency of 44100 Hz. As it is clear from Fig. 3b, the signal spectrum is smoothed and has gradual decrease with frequency increase. Such signals are called pink or gray noises. In the spectrum in Fig. 3b, there is a local maximum in the region of 18-21 kHz determined
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by receiver resonance. At low frequencies, there is an increase at the supply network frequency of 50 Hz.

During the growth of rock stress and deformation rate, increases of both pulse amplitude and of their number per time unit are observed. As an example, consider acoustic signal recorded on November 16, 2007. Fig. 3 illustrates a 10-minute fragment of signal record, and its energy spectrum is shown in Fig. 3d.

Fig. 3. Examples of acoustic signals during background period (a) and during deformation rate increase (c), and their energy spectra (b, d) respectively.

Comparison of the signal spectrum during disturbances (Fig. 3d) with the spectrum during the calm period (Fig. 3b) shows that significant increase of the signal level in the range of 1-18 kHz is observed. “Gray” or “pink” noise of the background period was changed by almost “white” noise of deformation disturbances. The signals (Fig. 3c) were observed on November 16 from 02:30 UT within 11.5 hours. In 1.5 of a day on November 17, 2007 at 17:16 UT an earthquake with the energy class of $K = 12.8$ occurred at the epicentral distance of 104 km. Hypocentre coordinates are 52.8°N, 159.63°E, the depth is 17 km (hereafter the data of Kamchatka Branch of GS RAS were used in the text; for energy classification of earthquakes, $K$ classes according to S.A. Fedorov scale were used; the relation of $K$ with magnitude $M_{LH}$ is determined by the formula $M_{LH} = (-4.6)/1.5$). The effect of weather conditions and industrial noises on the formation of acoustic emission signals was considered. We should note that it is not difficult to recognize additional disturbances of the emission under the influence of bad weather conditions according to meteorological observations, and industrial noises are classified easily [13].

Evaluation of acoustic emission directivity was carried out during background periods when there were no strong and long acoustic anomalies and during disturbances [11, 13]. Fig. 4 shows examples of diagrams of acoustic activity azimuthal distribution during
intensive disturbances determined by deformation changes in sedimentary rocks at the site at the background of their averaged values on calm days.

Fig. 4. Diagram of acoustic activity azimuthal distribution (dashed line) on December 14, 2007 (a) and on May 14, 2008 (b). Solid line is the background acoustic activity in November 2007 – February 2008(a), in May – June 2008 (b). Arrows indicate the directions from earthquake epicenters

Usually, space distribution of acoustic activity is quite isotropic in the absence of disturbances. In the both cases shown in Fig. 4, increased activity is registered from the South-East and the North-West (solid line). The structure of other lobes is repeated to a large extent. Differences in emission activity have seasonal character. According to the large number of irregularly arranged lobes of the emission directivity diagram in Fig. 4, we may judge on the complicated character of stresses, and according to the asymmetry of arrangement of radiation maxima, we may conclude on the inhomogeneity of medium properties around the observation point. At the background of smooth seasonal variations of acoustic emission, short-time (within a day) intensive disturbances occur. In Fig. 4, acoustic activity $D(\alpha, t)$ during these periods is shown by a dashed line. An example of anisotropy of acoustic emission directivity registered on December 14, 2007, a day before the earthquake with the energy class of $K=11.6$ which occurred on December 15, 2007 at 9:00 UT at the epicentral distance of 175 km in the azimuth of 114°, hypocentre coordinates are 52.34°N, 160.61°E, is shown in Fig. 4a. Emission anomaly lasted for seven hours, from 3:00 to 10:00, on December 14, 2007. In this case the highest activity of pulses was observed from the directions of 10 – 20 degrees. Moreover, somewhat smaller increase of activity was also registered from the direction range of 60 – 80 degrees. For comparison, the solid line shows the averaged background activity for the period from November 2007 to February 2008. Fig. 4b shows an example of anisotropy of acoustic emission directivity registered on May 14, 2008, a day before the earthquake with $K=11.1$ which occurred on May 15, 2008 at 5:49 at the epicentral distance of 127 km in the azimuth of 104°, hypocentre coordinates are 52.7°N, 160.06°E. The emission anomaly lasted of eight hours, from 0:00 to 8:00, on May 14, 2008. The highest activity of pulses was observed in the direction of 30°. Moreover, somewhat smaller activity increase was also registered from the direction range of 330 – 340 degrees. For comparison, the solid line shows the averaged background activity for the period from May to June 2008.

In spite of the fact that both earthquakes occurred in the azimuth of 100 – 115 degrees relatively the site, anomalous increases in pulse activity in the directions close to 15 – 30 degrees were registered before the events, though the graphs slightly differ
on the whole. We should note that none of the active regions corresponds to the direction of earthquake epicenter.

To confirm the deformation nature of the detected high-frequency anomalies of acoustic emission, simultaneous observations of rock emission and deformation were carried out. A laser unequal-arm strainmeter-interferometer constructed at TOI FEB RAS was used to measure relative deformations. It was installed at the distance of 50 m from the acoustic system on casing pipes of two 5-meter dry wells located at the distance of 18 m from each other.

On one of the wells we put an interference unit with a frequency-stabilized helium-neon laser covered by a box. On the other is an angle reflector protected by a container. Instability of laser frequency for a day is not more than $2 \times 10^{-9}$, radiation wavelength is 0.63 \(\mu\)m, measurement frequency is 860 Hz. When a 14-bit AD converter is used, strainmeter sensitivity is about $4 \times 10^{-11}$ m, and the measurement accuracy for relative deformations is about $2 \times 10^{-12}$. Of course, when a strainmeter is installed on the ground surface, such accuracy of measurements may not be realized without a special cover. Taking into account the vibration and weather condition effect at the point of observations, the measurement accuracy for relative deformations was about $10^{-8}$. The acoustic observation data were compared with rock relative deformations $\varepsilon$ and estimates of deformation rate $\dot{\varepsilon}$ calculated as the first differences of $\varepsilon$ measurements averaged on 1-second interval [14].

The results of joint investigations of acoustic emission and deformations showed that high-frequency anomalies of the emission were observed both during near surface rock tension (Fig. 5b) and compression (Fib. 5b) with relative deformation for a day of $10^{-7}$, and in a number of cases $10^{-6}$ (Fig. 5) during significant increase of deformation rate. When comparing the graphs of the emission and deformations, it is clear that acoustic disturbances occur during multiple shifts of near surface rocks of different amplitude. Deformations of some shifts are small, even for comparatively large amplitude they are not more than $10^{-8}$ (Fig. 5). The data illustrated in Fig. 5 were obtained during seismically calm periods when no earthquakes with K>10 were registered at the distance up to 250 km.

During the final stage of earthquake preparation, the effect of deformations on acoustic emission behavior is the most vivid [14]. Fig. 6 illustrates an example of simultaneous acoustic emission anomaly and rock deformation registered on May 1, 2007, 25 hours before the earthquake with the energy class of 12.1 which occurred on May 2, 2007 at 12:00 UT at the epicentral distance of 154 km. Hypocentre coordinates are 52.44°N, 160.33°E, the depth is 12 km. It is clear from the figure that from 1:00 to 9:00 quite a sharp compression of rocks was observed. It was followed by relieves from 1 to 5 minutes long accompanied by deformation rate increase and simultaneous rise of emission level in kilohertz frequency range. Compression amplitude reached 0.025 \(\mu\)m, and deformation rate increased up to $10^{-9}$ s$^{-1}$.

To estimate the relation between acoustic emission and rock deformations, we calculated cross-correlation functions (CCF) between acoustic pressure $P$ series in the range of 2.5 – 6.5 kHz and relative deformation , as well as the deformation rate $\dot{\varepsilon}$ from 0:00 till 12:00 on May 1. The sampling frequency of all the series was reduced to 0.25 Hz. In both cases the CCF maximum was observed at zero shift and it was –0.53 and 0.42 respectively for the significance level of no less than 0.001 [14].
Fig. 5. Examples of acoustic emission anomalies and deformations during near surface rock tension on October 14, 2009 (a), and near surface rock compression on October 18, 2009 (b). \( \varepsilon \) is rock relative deformation, \( \dot{\varepsilon} \) is deformation rate, \( P \) is acoustic pressure accumulated over the 4-second period in the frequency range of 0.6 – 2.0 kHz.

Fig. 6. Example of acoustic emission anomaly and rock deformations before the earthquake on May 2, 2007 at 12:00 UT. \( P \) is acoustic pressure accumulated over the 4-second period in the frequency range of 2.0 – 6.5 kHz. Other symbols are the same as in Fig. 5.
Conclusion

We ascertained the existence of acoustic emission effect in sedimentary rocks which consists in the increase of geoacoustic radiation intensity in the frequency range from hundreds of hertz to the first tens of kilohertz during rock mass deformation rate increase. The effect has been stably observed during more than 10-year natural experiment at several spaced sites in Kamchatka, and it occurs the most intensively during the final stage of earthquake preparation.

During the acoustic emission effect, a clearly defined anisotropy of geoacoustic radiation directivity occurs. It is determined by the source orientation of acoustic oscillations in the stress field of near surface sedimentary rocks.

References


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SPECIAL ASPECTS OF CALIBRATION OF IONIZING RADIATION DETECTORS USED FOR SOIL RADON MONITORING

V.S. Yakovleva¹, P.M. Nagorskiy²

¹ National Research Tomsk Polytechnic University, 634050, Tomsk, Lenin st., 30 Russia
² Institute of Monitoring of Climatic and Ecological Systems SB RAS, 634055, Tomsk, Akademicheskaya st., 10 / 3., Russia
E-mail: vsyakovleva@tpu.ru

The results of calibration of α-, β- and γ-radiation detectors mounted into a borehole at the depths of 0.5 and 1 m, which are destined for soil radon monitoring, are represented and analyzed. The radon isotope radiometer RTM 2200 (SARAD GmbH, Germany) was used for calibration. It was determined that time variations of α-radiation flux density at the depths of 1 m poorly reflect the soil radon dynamics, both the diurnal variations and their amplitudes, and in the case with γ-radiation, they do not reflect it at all. Good synchronism between flux density dynamics of β-radiation at 0.5–1 m depth that of α-radiation at 0.5 m depth and radon volumetric activity dynamics measured at the same depths was found for diurnal and synoptic scales. Nevertheless, for certain days a small time shift between α- and β-flux densities and radon time series was observed. Recommendations for the conditions and the procedure of calibration of soil ionizing radiation detectors were developed.

Key words: radon, soil, monitoring, detector, ionizing radiation, calibration

Introduction

Methods of control for soil radon on α-, β- or γ-radiation in boreholes are the most widely spread to make earthquake forecast, to investigate the issues of gaseous exchange between lithosphere and atmosphere [1]-[9]. In these methods, ionizing radiation detectors operating in the count mode are mounted into boreholes at some depth. In this case, the processes of gas transfer in soil are not disturbed in comparison to the methods with forced air pumping from a borehole for the following analysis.

The reasons of the change of radiometers for α-, β- or γ-radiation detectors, mounted into boreholes are the simple maintenance and the capability of remote automated continuous monitoring of soil radon. Moreover, they are 1-2 orders cheaper than the...
methods applying $\alpha$-spectrometry which makes it possible to use them widely. Thus, the number of sites for simultaneous monitoring may be significantly increased, extending the observation area. The main advantage is that they allow us to receive, process and analyze data in quasi-real scale.

Validity of the results and reliability or the methods of radon direct registration in a borehole applying ionizing radiation detectors has not been investigated in detail and over long periods of time. The conversion of pulse counting rate measured by detectors into radon volumetric activity (VA) value is carried out by multiplication by a correction coefficient which is usually determined by an indirect method of comparison of the data with the results of single (in the best case, not continuous) measurements by a certified radiometer for radon.

Frequently, the analysis of soil radon dynamics for “anomalies” to make earthquake forecast is performed according to the results of monitoring without the conversion of the measured pulse counting rate into the radon VA value, arguing that only relative variations are important in such tasks.

Nevertheless, analysis of the results of numerical modeling [8, 9] allowed us to suppose that for the conversion of ionizing radiation (IR) flux densities (FD) the correction coefficients will not be proportional to radon VA in the soil air, and will likely be the functions from one or several parameters. Moreover, the conversion of measurement results into absolute values may be useful in the following calculation of radon transfer in soil-atmosphere system and in specification of model coefficients, as well as in the study of the issues on gaseous exchange, air mass and air electricity transfers and so on.

To verify the possibility of radon measurement by direct registration in the soil (borehole) by one or several IR and to determine the correction coefficients, a series of long-term calibration experiments has been carried out in Tomsk laboratory of radioactivity and ionizing radiations (TORII).

Instrumentation and methods

To monitor soil radon, highly sensitive intellectual detector blocks, BDPA-01 (2 units), 2 BDPB-01 (2 units) and BDKG-03 (ATOMTEX, Belarus), where chosen. They were installed on TOPII experimental test field, in 5 separate boreholes 0.5 and 1 m deep according to the scheme shown in Fig. 1a. The external view of the complex for the calibration of soil detectors of ionizing radiation by radon and thoron radiometer RTM 2200 (SARAD, Germany) is shown in Fig. 1b. The calibration of soil detectors was carried out from May 28 to July 28 and from October 5 to November 21, 2011.

To realize the calibration, 2 holes were made in the upper part of a plastic pipe (Fig. 1b) projecting above the surface to connect the tubes with the radiometer. The air together with gases cumulated inside a borehole were constantly pumped out (with the velocity of 1 l/min) by a built-in air pump via the 1st connecting tube from the lower part of a borehole and got inside the test radiometer volume. During the time of gas motion from a borehole into the radiometer, thoron almost decayed. Thus, the developed scheme allowed us to analyze only soil radon. After the test volume, the air together with radon was pumped back into the upper part of a borehole through the 2nd connecting tube.
Results and analysis

Let's first analyze the differences in the regions for data collection of \(\gamma\)-radiation photons, \(\beta\)-particles and \(\alpha\)-particles formed during radio active decay of soil radionuclide, radon isotopes and their daughters (D). Consider the schemes of mounting of detectors for photons (Fig. 2 a) and charged particles (Fig. 2 b and c). It follows from the schemes that the regions for charged particles and photon collection differ significantly.
Collection of $\gamma$-radiation photons by a detector takes place within the radius of 50-60 cm depending on the borehole size. Taking into account that the borehole for $\gamma$-detector is hermetically sealed at the bottom, the detector registers only the photons generated in the soil. Collection of $\alpha$-particles is performed from a small air volume limited by plastic pipe walls with an open base and the distance to soil corresponding to the length of maximum range of $\alpha$-particle path in the air determined by the size of detector sensitivity area, maximum $\alpha$-particle energy (8.8 MeV) and energy threshold of scintillation detector (3 MeV).

$\beta$-particles get into the detector from the same air volume plus from the soil layer about 0.5 cm thick. The change of weather conditions causes the change of radon concentration in the soil. In this case, time delay of the reaction response of the whole “soil-air inside a borehole” system or its separate parts is possible. It will be determined by the differences in time of radioactive equilibrium between $\alpha$, $\beta$-radiating decay products.

Thus, the differences in the regions for collection of IR, characterized by different sizes and characteristics of included media, allow us to expect differences in the dynamics of different types of IR characterizing the changes in soil radon concentration.

The results of calibration in the borehole at the depth of 1 m with $\alpha$-detector are shown in Fig. 3. The upper diagram shows experimental series of radon VA (Rn 1 m) and the radon VA (Alpha 1 m) recovered by multiplication by the correction coefficient $K_{\alpha1m}$. The lower diagram of the figure illustrates temperature changes on the earth surface.

![Fig. 3. Dynamics of temperature and radon VA at the depth of 1 m](image-url)
The correction coefficient $K_{\alpha_{1m}}$ was determined by the division of radon VA average for the observation period by the average pulse counting rate registered by the detector.

Analysis of the real and recovered series of radon VA showed that the $\alpha$-detector at the depth of 1 m reflects inadequately the real change of radon VA in a borehole, there are no diurnal variations which are registered by radon radiometer.

Then the calibration of $\alpha$-detector was carried out in the borehole at the depth of 0.5 m from June 3 to 10. The results of calibration in the borehole 0.5 m deep with $\alpha$-detector are illustrated in Fig. 4.

![Fig. 4. Results of calibration by $\alpha$-detector in the borehole 0.5 m deep](image)

It shows the series of radon VA (Rn 0.5 m) measured by the radiometer and those (Alpha 0.5 m and Beta 0.5 m) recovered by the multiplication by the calculated correction coefficients, as well as temperature on the earth surface.

Recovery of $\beta$-series at 0.5 m by a simple multiplication by the correction coefficient failed. As the result, the correction coefficients for the conversion of measurement data from pulse/s into radon VA units (Bk/m³) were determined for $\alpha$- and $\beta$-detectors by different ways. A more complicated scheme was required with the recovery of $\beta$-series to determine the correction coefficient $K_{\beta}$. The pulse counting rate values for $\beta$-detector ($N_\beta$) were divided into 2 parts: a) the constant one ($N_{\beta s}$), determined by soil radionuclide not referring to radon component and b) the variable one ($N_{\beta Rn}$), determined by $\beta$-radiating radionuclide of radon decay chain which are contained in the borehole air and the 5 cm soil layer in the bottom open borehole basis.

Thus, the total rate of pulse counting was determined as $N_\beta = N_{\beta s} + N_{\beta Rn}$, and radon VA was estimated by the expression

$$VA_{Rn}(i) = (N_\beta(i) - N_{\beta s}) \cdot K_{\beta}.$$

The constant component $N_{\beta s}$, which value was 47% from the average pulse counting rate for the borehole under the study depend on physical-geological characteristics of the area under investigation, distance from the detector to the earth surface and borehole diameter.
Analysis of calibration results for the borehole 0.5 m deep with $\alpha$-detector showed that temporal changes of radon VA and $\alpha$- and $\beta$-radiation fluxes measured at one depth of 0.5 m but in different boreholes are almost synchronous, they have almost a saw-tooth form. Amplitude of $\alpha$-radiation FD variations changes in time according to radon field. Dynamics of $\beta$-radiation FD at the same depth almost coincide with $\alpha$-field radon dynamics for the period under consideration.

Time variations of radon VA, FD of $\alpha$- and $\beta$-radiations at the depth of 0.5 m correlate well with the dynamics of the earth surface temperature.

To estimate how the system of air pumping during the calibration affects the behavior of $\alpha$- and $\beta$-fields, Fig. 3 illustrates a longer period of about 2 days before and after the calibration. When the system of air pumping was connected via the radon radiometer, the average $\alpha$-background decreased by 20% at the depth of 0.5 m. Variation amplitude did not significantly change. Such a reaction of the $\alpha$-background is associated with the partial removal of thoron and $\alpha$-radiating decay products of radon and thoron from the air in the borehole. When the air pumping was stopped, $\alpha$-background in the borehole quickly recovered.

The system of air pumping affected the $\beta$-background more significantly. On June 10, the system of air pumping with RTM 2200 radiometer was connected to the borehole 0.5 m deep with a mounted $\beta$-detector inside. Variation amplitude decreased by almost two times (Fig. 4).

The calibration results for $\beta$-detector in the borehole 0.5 m deep are shown in Fig. 5. The upper diagram illustrates the series of radon VA in the borehole 0.5 m deep (Rn 0.5 m) measured by the radiometer and the series of radon VA measured $\beta$-radiation and recovered by multiplication by the correction coefficient (Beta 0.5 m). For comparison, the lower diagram of Fig. 5 shows the series of radon VA measured $\alpha$-radiation and recovered by multiplication by the correction coefficient (Alpha 0.5 m).

![Fig. 5. Results of calibration of $\beta$-detector in the borehole of 0.5 m](image)

It was obtained that the dynamics of $\beta$-radiation FD at the depth of 0.5 repeats well the radon dynamics at the same depth in contrast to $\alpha$-radiation FD measured in the neighboring borehole at the distance of only 1.5 m.
It was also obtained in this calibration experiment that α-radiation FD at the depth of 0.5 m does not quite correctly reflect the soil radon dynamics, both the diurnal variation and the variation amplitude. During some periods, radon diurnal variation advance up to asynchronous is observed (from June 14 to 17, 2011).

The last calibration procedure of the summer season was that of β-detector in a borehole 1 m deep. During the calibration, anomalous increase of radon VA at the depth of 1 m was observed (Fig. 6) from July 13 to 15 which, as analysis showed, was caused by intensive precipitation (47 mm). At the same time almost synchronous anomalous increases of α-radiation flux density at the depths of 0.5 and 1 m and that of β-radiation at the depth of 1 m were registered.

![Fig. 6. Results of calibration of β-detector in the borehole 1 m deep](image)

Recovery of radon VA series according to β-detector data at the depth of 1 m was carried out by the same scheme as it was described above. Good agreement with the values measured by radon radiometer was obtained. Nevertheless, during some periods, the advance of β-background diurnal variation at the depth of 1 m in comparison to radon VA by about 2-3 hours was observed. Diurnal variation of α-radiation FD at the depth of 1 m was not determined during the calibration, thought radon VA at the same depth showed clear diurnal variations. Thus, we may conclude that α-field at the depth of 1 m does not reflect diurnal variations of radon VA, thought it reacts on strong external effects but with less sensitivity. The diurnal variations of α- and β-radiation FD at the depth of 0.5 m showed almost 6-hour delay in the maxima in comparison to radon VA variation at the depth of 1 m.

Analysis of calibration results showed that temperature diurnal variation for the observation period may be described by the function $T(t) = T_0 + T_1 \cos(w(t - t_m))$, where $T_0=18.9$ °C is the average temperature, $T_1=4.78$ °C is the maximum deviation from the average value $w=0.261799$ is the frequency, $t_m$ the time of maximum which was 17:00. The radon field is well described by a similar function with 2 hour delay of the maximum, i.e. $t_m=19$ h. Temperature variation within a day were about 25%, and radon VA was 18%. If we suppose that temperature change is the only source of radon field variations, increase in temperature by 1 °C would cause radon VA increase (at the depth of 0.5 m) by 300 Bk/m³, and vice-versa respectively.

The procedure of detector calibration was partially repeated in autumn 2011. Special consideration was given to the questions of borehole pumping effect since due to the big sizes of "etalon" radon radiometers, calibration of soil detectors of ionizing radiations...
may only be realized by the method of air pumping out from a borehole for the following analysis.

The obtained during the autumn calibration experiment series of radon VA measured by a radiometer in a borehole 1 m deep (Rn 1 m) and the series of radon VA of α- and β-radiations at the depth of 1 m recovered by the multiplication by the correction coefficients are shown in Fig. 7. α-radiation FD dynamics at the depth of 1 m repeats well the dynamics of radon VA at the same depth in contrast to β-radiation FD. However, strong effect of the process of cyclic air pumping through the borehole and the radiometer on the value and the dynamics of α-radiation flux density in the calibrated borehole was discovered.

![Fig. 7. Radon VA dynamics at measured by the radiometer at the depth of 1 m and the recovered series of radon VA at the depth of 1 m according to β- and α-radiations](image)

The counting rate of pulses from α-radiation in the borehole decreased by almost 4 times after the connection to the scheme of cyclic blowing. At that, standard deviation significantly increased. Though when the borehole pumping was over, α-background (in radon VA units) increase and equaled to β-background.

Strong effect of the pumping system during the autumn calibration experiment in contrast to the summer experiment, when after the connection of the pumping system the average α-background in the borehole of 0.5 m deep decreased by only 20%, may be explained by seal failure at the points of connection of tubes with the plastic pipe casing the borehole due to larger temperature differences.

Analysis of the calibration experiments showed the following. Some correction coefficients, determined for one detector in repeated experiments had strong deviations. For the β-background it may be explained by the fact that when the air pumping system was connected, the detector was moved to another distance from the surface which caused the change of \( N_{\beta s} \) value changing from 2.1 to 3.1 pulse/s (from 47% to 65% from the total counting rate of pulses). Thus, the change of β-detector mounting height in the borehole significantly affects both the correction coefficient for the conversion of the counting rate into the volumetric activity and the constant \( (N_{\beta s}) \) component of counting rate determined by β-radiating soil radionuclide not referring to DDP of radon and thoron.

It was also obtained that during the calculation of the correction coefficients for the conversion of the measured value into radon VA values, we should consider the fact for the soil air that forced air pumping out of a borehole decreases the flux of β- and α-radiations. The range of diurnal variations reduces significantly in this case. Analysis of real and recovered series of radon VA showed the following.
α-radiation FD at the depth of 1 m reflect inadequately the real change of radon VA in the borehole, there are no diurnal variations. Agreement of radon VA and α-radiation FD at the depth of 1 m was registered only in autumn. Mainly, α-radiations reflect only the changes of averaged over 1-2 days values of radon VA with the error of 30%. However, during anomalous emissions of radon, α-radiation FD at the depth of 1 m evidently reacts which makes this parameter acceptable to predict hazard phenomena, for example, the change of the stress-deformation state of the Earth crust but with some limitations.

α-radiation FD at the depth of 0.5 m shows time delay of the moments of maxima in radon VA in comparison to the depth of 1 m. The delay may reach 8 hours. According to the time delay, radon motion velocity in the soil was estimated to be $17 \times 10^{-4}$ cm/s, that is almost 3 times higher than radon motion velocity only via molecular diffusion, $6 \times 10^{-4}$ cm/s.

β-radiation FD at the depths of 0.5 and 1 m quite well reflects the soil radon dynamics. When air temperature is positive and it is not raining, diurnal variation is clearly observed. However, β-field diurnal variation has a shift during some periods in comparison to the diurnal variation of radon field, i.e. there are advances/delays by several hours in the time of maxima of β-field dynamics during various time periods.

γ-radiation FD at the depth of 1 m does not reflect the radon VA at the same depth. According to the results of calibration, recommendations for the procedure of soil detector calibration were developed.

**Recommendations for calibration of IR detectors to control soil radon**

Recommendations for the conditions and the procedure of calibration of ionizing radiation detectors mounted in boreholes and applied to control soil radon dynamics in monitoring regime may be formulated as follows.

- Applying the method of air pumping from a borehole during the calibration procedure, application of a borehole with already mounted detectors is not recommended. Another borehole with analogous characteristics (depth, diameter) should be drilled at the distance not less than 70 cm and not more than 3 m. Thus, the conditions of natural air exchange between the soil and the atmosphere will not be violated.

- If in the result of maintenance of boreholes with detectors the IR detectors were moved, removed or exchanged, calibration should be repeated and new calibration coefficients should be determined. β-scintillation detectors are very sensitive to movements.

- If calibration was carried out in boreholes with IR detectors mounted inside, besides the detection of the correction coefficient, it is also necessary to detect the correction for diurnal variation range decrease after air pumping out of a borehole. To do that, the data from IR detector should be registered several days before and after the calibration procedure.

- IR detectors should not be taken out of a borehole or moved during the calibration, since it results in data time series distortion.

- Boreholes with IR detectors mounted inside should not be opened during calibration, i.e. the tubes for air pumping from a borehole which are cyclically connected to radon radiometer should be mounted at least one day before the experiment which is enough to recover the radon equilibrium activity in a borehole.
Conclusions

Analysis of the results of calibration experiments showed that it is better to apply β-radiation detectors which can be mounted at the depth of 0.5-1 m to monitor soil radon. α-radiation detectors may be used to monitor radon only when mounted at the depth of about 0.5 m. γ-radiation detectors are not good for soil radon monitoring.

According to calibration results, recommendations to the conditions and the procedure of calibration of soil IR detectors were developed.

If radon monitoring is aimed at earthquake forecast or investigation of hazard natural phenomenon, the change of the parameter of radon VA measurement by β-radiation FD is quite reasonable and by α-radiation FD may cause great difficulties in interpretation of monitoring data, however, it may be applied with some limitations and assumptions.

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References


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THE DEVELOPMENT OF RADIATION MONITORING TECHNOLOGY FOR URBAN ENVIRONMENT

V.S. Yakovleva\textsuperscript{1}, P.M. Nagorskiy\textsuperscript{2}

\textsuperscript{1} National Research Tomsk Polytechnic University, 634050, Tomsk, Lenin st., 30 Russia
\textsuperscript{2} Institute of Monitoring of Climatic and Ecological Systems SB RAS, 634055, Tomsk, Akademicheskaya st., 10 / 3., Russia

E-mail: vsyakovleva@tpu.ru

The results of monitoring of meteorological and radiation parameters in Tomsk Observatory of Radioactivity and Ionizing Radiation are presented and analyzed in this work. The advantages of new radiation monitoring technology including the investigation of radiation parameters vertical profiles are presented. The existing models of soil and atmosphere radon isotopes transport were verified for urban environment according to the analysis results of radiation monitoring data. The verification showed that the existing models do not allow describing some experimentally obtained dependences and require further development. For instance, it was experimental obtained that alpha-radiation flux density increases for some times with height up to 25 m and then is practically unchanged up to 35 m. The same is for radon. The explanation of such dependence in radon behavior is the influence of urban infrastructure. It was also experimental obtained that diurnal variations of soil radon volumetric activity measured by alpha- and beta-radiation decrease with depth until fully amplitude losses, but the moments of maximums (minimums) occur with time delay. Such behavior is dependent on thermophysical properties of soil and local meteorological conditions. It was proposed to describe the soil gas advection velocity by a function considering the soil thermal conductivity and the dependence of soil temperature on depth.

\textit{Key words:} radiation monitoring, technology, ionizing radiation, radon, soil, atmosphere

Introduction

The notable indicator properties of radionuclides and ionizing radiation are known and have been actively applied for a long time to obtain knew knowledge on the dynamic processes occurring in the atmosphere and lithosphere, to upgrade the models of gas and aerosol transfer and to make forecast of hazardous phenomena of natural and anthropogenic character.
The role of ionizing radiation and natural radioactivity, especially of radon gas, is important in radio ecology, seismology, atmosphere near-ground layer physics and in construction work. Thus, research teams and state structures make radiation monitoring of the near-ground atmosphere and study the dynamics of activity of some radionuclides in the near-ground atmosphere and soil.

Unfortunately, artificial radioactivity caused by nuclear testing, industrial disasters and technological processes is still the main direction of radiation environment control. Thus, the special attention is paid only to γ-background monitoring. Variations of field characteristics for other kinds of ionizing radiations (α-, β-radiations) have not been controlled that was explained by their low penetrability and, correspondingly, low informative value. Monitoring of soil radon and radon flux density from the earth surface is carried out mainly to make earthquake forecasts and occasionally to make radio ecological and geo-ecological surveys before construction works.

Research team [1] developed a complex approach to radiation monitoring. The main “spice” of the technology was the investigation of the vertical profile of field characteristics for ionizing radiations (IR) and for radon in soil-atmosphere system. The aim of this work was to determine the advantages in the study of radiation parameters vertical distributions in comparison to the traditional approach when only one height (depth) or one radiation parameter is under investigation.

One of the tasks of the study is the development of a technique for subject processing of meteorological and radiation parameter monitoring archive data which is determined by the application area of the results: radiation ecology and biology; construction works; seismology or atmosphere physics. The results of radiation monitoring were used to verify the existing models of radionuclide transfer and the peculiarities of ionizing radiation transfer for urban environment as well.

**Instrumentation and methods**

Radiation monitoring at Tomsk Observatory of Radioactivity and Ionizing Radiation has been carried out since 2008 and its technology is constantly upgrading. At present, the radiation monitoring includes synchronous continuous automated high sampling rate (1-10 min) measurements of IR field characteristics (α-, β-, and γ-radiations), radon and thoron flux densities (RFD and TFD) from the soil surface, and volumetric activity (VA) of radon, thoron and daughter products of their decomposition (DPD) at the depths up to 5 m and the heights up to 35 m. The scheme of monitoring of ionizing radiation field structure and dynamics and natural radioactivity in the near-ground atmosphere and in the soil surface layer is shown in Fig. 1.

The complex approach to radiation monitoring allowed us to obtain new important scientific findings. In contrast to the traditional assumptions, a new dependence of radon volumetric activity (VA) (alpha-radiation flux density) on height above the ground surface was discovered (Fig. 2).

Monitoring of atmospheric-electric and meteorological parameters is carried out simultaneously at the Institute of Monitoring of Climatic and Ecological Systems SB RAS (Akademgorodok, Tomsk) [1].

Traditional models show exponential decrease of radon volumetric activity with height. Our results showed that alpha-radiation flux density (FD) increases by some times with height up to 25 m and then is practically unchanged up to 35 m.
Fig. 1. Scheme of monitoring of IR field structure and dynamics and natural radioactivity in the near-ground atmosphere and in the soil surface layer.

Fig. 2. Dependence of alpha- and beta-radiation flux densities on height.
The same is for radon. The explanation of such dependence in radon behavior is the influence of urban infrastructure. There are high buildings around the testing site which may affect the turbulent processes and air mass transfer. Moreover, some of the detectors (Fig. 1) are located on a mezzanine of IMCES building.

Furthermore, radon VA increase with height should also be taken into account when simulating radon level inside the rooms of multistory buildings since airing may cause not radon level decrease, according to traditional assumptions, but to its increase. As for the widely known model of radon transfer in the soil, considering only diffusion transfer, it is meant for averaged over a long period (several days and more) characteristics of RFD or radon VA in the soil to estimate the radon potential of territories, but it is not meant for intra-day variation simulation.

Diffusive-advection model of radon isotope transfer in the soil [1, 2] has a wider application and allows us to describe the majority of experimental data but it does not lack some disadvantages since the advection velocity is the constant value. This assumption does not enable us to describe experimentally obtained data series on radon soil field characteristics having diurnal and seasonal variations. Moreover, our research team discovered that the times of maxima (minima) in diurnal variations (Fig. 3 and 4) depend on the depth of detector position and correlate with soil temperature changes at different depths. The X-axes in Fig. 3 and 4 show the number of a day of the year.

Analysis of the experimental data obtained at Tomsk Observatory of Radioactivity and Ionizing Radiation showed that improvement of both radionuclide transfer model for urban atmosphere and of soil radon isotope transfer model is required by the change of model constant coefficients by approximation functions determined on the basis of experimental data.

The advection velocity should be described by the function of soil depth and time. Experimentally obtained diurnal variations of radon VA decrease with depth and the moments of maximums (minima) occur with time delays. Such behavior is dependent on thermophysical properties of soil and local meteorological conditions. When there is no precipitation, diurnal and seasonal temperature changes affect the radon VA diurnal variations the most.

Annual and diurnal temperature changes may be described by a sinusoidal function of time and depth [4, 5]. Thus, the expression for advection velocity function is upgraded as follows:

\[ v(x, t) = \frac{A_0 v_0}{T_a} \sin (\omega t + \phi(z) - z/d) \exp (-z/d), \]

where \( v_0 \) is the average advection velocity experimentally determined for soil layer of 1 m thick, m/s; \( T_a \) is the average surface temperature, °C; \( A_0 \) is the temperature change amplitude on the soil surface, °C; \( \omega \) is the radial frequency; \( \phi(z) \) is the delay time (initial phase) in \( z \) function, s; \( d = \sqrt{2k/(C\omega)} \) is the depth of temperature change attenuation in the soil, i.e. at this depth temperature amplitude decreases by \( e^{-2.718} \) times in comparison to that on the soil surface \( (A_0) \), m; \( k \) – thermal conductivity coefficient, J/m·°C; \( C \) is the volumetric heat capacity, J/m·°C·°C. This presentation of advection velocity allows us to describe the influence of temperature and thermal conductivity on radon VA in diurnal and annual variation. Thus, to make numerical simulation of the structure and variations of both atmospheric radiation background and radon transfer in the soil, it is necessary to know and to control
a number of meteorological values (pressure, temperature, wind velocity, atmosphere turbulence and precipitation characteristics and so on).

Fig. 3. Diurnal variations of soil temperature and radon VA in the soil measured by: alpha-radiation and beta-radiation at different depths
Conclusions

Further development of radiation monitoring technology for urban environment is the upgrading of the models for gas and radionuclide transfer in different environments, construction of new algorithms and methods of control of both radiation values and meteorological atmospheric-electrical and actinometric values. It allows us to obtain new data on the structure and dynamics of ionizing radiation fields and natural radioactivity in the near-ground atmosphere and near surface soil layer, to determine the features and regularities in their behavior and the relations with meteorological processes of intra-day, diurnal and synoptic scales.

One of the results of the monitoring is the constantly refilling data library, including the data bases on the characteristics of radiation fields and radionuclide VA in the soil and in the near-ground atmosphere, atmospheric-electrical and meteorological values, on the reoccurrence and intensity of extreme events associated with meteorological phenomena in a seismically safe region with sharply continental climate in the conditions of the current changes. The data bases and monitoring results may be useful in Rospotrebnadzor, Rosgidromet, Federal Rescue and Public Health Services as well as in scientific organizations which make forecasts of change of stress-strain state of the Earth crust.

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ABOUT THE AUTHORS


Kuznetsov Vladimir Valeryevich – Dr. Sci. (Tech.), Leading Researcher comprehensive geophysical observatory «Paratunka», Institute of Cosmophysical Researches and Radio Wave Propagation FEB RAS.

Marapulets Yuri Valentinovich – Ph.D. (Tech.), Associate Professor, Deputy Director for Science Institute of Space Physics Research and Radio Wave Propagation FEB RAS, Professor of Dep. computer science Vitus Bering Kamchatka State University.

About the authors

Parovik Roman Ivanovich – Ph.D. (Phys. & Math.), Head of the Dep. Mathematics and Physics, Vitus Bering Kamchatka State University, Senior Researcher of Lab. Modeling of Physical Processes, Institute of Cosmophysical Researches and Radio Wave Propagation FEB RAS.

Yuldasheva Asal Victorovna – Ph.D. (Phys. & Math.), Lecturer of the Dep. Differential Equations and Mathematical Physics, of the National University of Uzbekistan, Tashkent.

Yakovleva Valentina Stanislavovna – Dr. Sci. (Tech.), Associate Professor, Professor of Dep. Applied Physics, National Research Tomsk Polytechnic University, Tomsk.
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