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Calculation of Atmospheric Electric Fields Penetrating from the Ionosphere

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Abstract—The spatial distributions of electric fields and currents in the Earth's atmosphere are calculated. Electric potential distributions typical of substorms and quiet geomagnetic conditions are specified in the ionosphere. The Earth is treated as a perfect conductor. The atmosphere is considered as a spherical layer with a given height dependence of electrical conductivity. With the chosen conductivity model and an ionospheric potential of 300 kV with respect to the Earth, the electric field near the ground is vertical and reaches 110 V m^{-1}. With the 60-kV potential difference in the polar cap of the ionosphere, the electric field disturbances with a vertical component of up to 13 V m^{-1} can occur in the atmosphere. These disturbances are maximal near the ground. If the horizontal scales of field nonuniformity are over 100 km, the vertical component of the electric field near the ground can be calculated with the one-dimensional model. The field and current distributions in the upper atmosphere can be obtained only from the three-dimensional model. The numerical method for solving electrical conductivity problems makes it possible to take into account conductivity inhomogeneities and the ground relief.

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1. ATMOSPHERIC CONDUCTOR MODEL

Large-scale atmospheric electric fields and currents with characteristic variation times longer than 10 min can be treated as static fields and currents in a conductor [Park, 1976]. If the spatial distribution of the electrical conductivity σ is known, the electric field **E** and current density **J** can be determined by solving the system of the electrical conductivity equations

$$\mathrm{div}\,\mathbf{J} = 0,\quad \mathrm{curl}\,\mathbf{E} = 0,\quad \mathbf{J} = \sigma\mathbf{E}, \tag{1}$$

which consists of the charge conservation law, induction equation, and Ohm's law, respectively. The second equation allows for the introduction of an electric potential V such that $\mathbf{E} = -\mathrm{grad}\,\mathbf{V}$ and the reduction of system (1) to the electrical conductivity equation

$$-\mathrm{div}(\sigma\,\mathrm{grad}\,V) = 0. \tag{2}$$

Figure 1 shows the typical altitude distribution of the atmospheric conductivity [*Handbook* ..., 1965]. The typical conductivity of the near-ground layer is 3×10^{-14} $(\Omega\ \mathrm{m})^{-1}$, which is much lower than the conductivity of sea water 3 $(\Omega\ \mathrm{m})^{-1}$, moist soil 0.01 $(\Omega\ \mathrm{m})^{-1}$, and even marble 10^{-8} $(\Omega\ \mathrm{m})^{-1}$. For this reason, the ground is usually treated as a perfect conductor, which corresponds to the boundary condition

$$V|_{r = R_E} = 0, \tag{3}$$

where R_E is the Earth's radius and r, θ, and φ are the spherical coordinates.

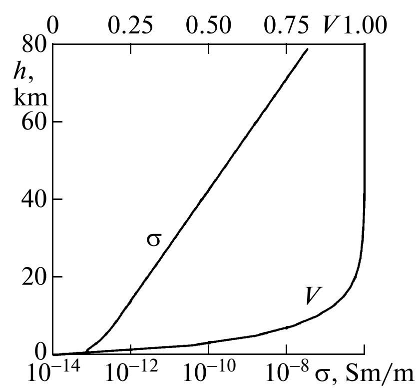

Fig. 1. Altitude distribution of the atmospheric conductivity σ from [Handbook ..., 1965] and the one-dimensional solution of the electrical conductivity problem *V*.

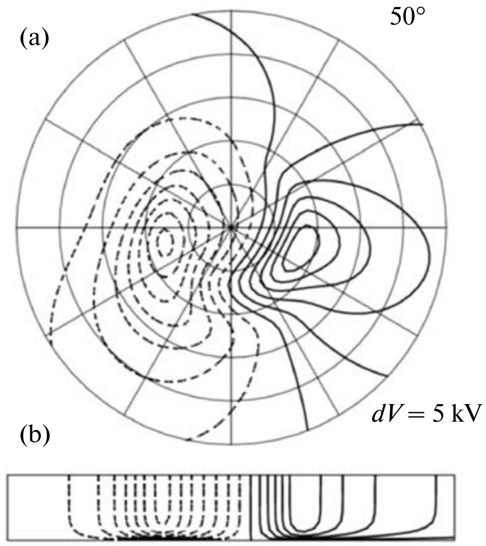

Fig. 2. Distribution of the electric potential in the atmosphere under quiet geomagnetic conditions: (a) the potential in the ionosphere $V_{\text{ion}}(\theta, \varphi)$ (the upward direction corresponds to the sunward direction) and (b) the potential V in the vertical plane passing through the points of the minimum and maximum of $V_{\text{ion}}(\theta, \varphi)$. The step between the neighboring equipotential lines is 5 kV. The dashed lines show negative potential values.

As compared to the atmosphere, the ionosphere is also a good conductor; hence, the potential distribution in it can be considered as given

$$V|_{r = R_E + H} = V_{\text{ion}}(\theta, \varphi), \tag{4}$$

where H is a certain altitude, $V_{\text{ion}}(\theta, \varphi)$ is a given function, which is constructed using the measurement results or is determined from the solution of the problems of the ionospheric electrical conductivity, where the atmosphere is considered as a perfect insulator [Denisenko and Zamay, 1992].

2. NUMERICAL METHOD FOR SOLVING THE ELECTRICAL CONDUCTIVITY PROBLEM

The solution of the boundary value problem specified by Eqs. (2)–(4) is equivalent to the determination of the minimum of the energy functional

$$W(V) = \int \sigma(\text{grad}\, V)^2 r^2 \sin\theta\, dr d\theta d\varphi \tag{5}$$

on the functions V satisfying conditions (3) and (4) [Mikhlin, 1957]. The $W(V)$ value for the exact solution V is the Joule dissipation. This variational principle makes it possible to construct the finite element method for numerically solving the problem. The calculation region is divided into elementary tetrahedra and piecewise-linear approximating solutions of the function V_h are considered. The V_h values at the boundary nodes are determined by conditions (3) and (4). The system of linear algebraic equations for V_h values at the inner nodes is obtained as the set of the conditions of the minimum of $W(V_h)$ values at each of the nodal values V_h. The matrix of this system is symmetric and positively definite under the natural constraints for the used grid [Mikhlin, 1957]. The system is solved using the Fedorenko multigrid method.

If the conductivity σ depends only on the altitude, $\sigma = \sigma(r)$, and the ionospheric potential is constant, $V_{\text{ion}} = V_0$, the problem specified by Eqs. (2)–(4) becomes one-dimensional:

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\sigma(r)\frac{dV(r)}{dr}\right) = 0,$$
$$V|_{r = R_E} = 0, \quad V|_{r = R_E + H} = V_0. \tag{6}$$

Figure 1 shows the solution of this problem. In the figure scale, it is indistinguishable from the solution of this problem disregarding the spherical shape of the Earth, when the equation has the form

$$-\frac{d}{dr}\left(r^2\sigma(r)\frac{dV(r)}{dr}\right) = 0. \tag{7}$$

This one-dimensional solution is used to construct the initial approximation V_h as a function for interpolating the potential along the vertical direction from zero to $V_{\text{ion}}(\theta, \varphi)$.

The problem specified by Eqs. (2)–(4) with $\sigma = \sigma(r)$ is also reduced to the ordinary differential equation if $V_{\text{ion}}(\theta, \varphi)$ is a spherical function. By means of the expansion of $V_{\text{ion}}(\theta, \varphi)$ into the spherical functions, sufficiently general solutions of the problem can be analytically constructed. We use them to test the numerical method.

3. ATMOSPHERIC ELECTRIC FIELDS PENETRATING FROM THE QUIET IONOSPHERE

Global models of the ionospheric electric fields created by longitudinal currents in the auroral zone during magnetospheric substorms and under quiet geomagnetic conditions were developed in [Denisenko and Zamay, 1992]. Here, we consider the contribution from these fields to the atmospheric electric field under quiet conditions. The results for disturbed conditions are similar.

Figure 2a shows the distribution of the electric potential $V_{\text{ion}}(\theta, \varphi)$ from [Denisenko and Zamay, 1992]. The numerical solution of the electrical conductivity problem given by Eqs. (2)–(4) provides the spatial distribution of $V(r, \theta, \varphi)$. Figure 2b shows the distribution of V in the vertical plane passing through the points of the minimum and maximum of $V_{\text{ion}}(\theta, \varphi)$ seen in Fig. 2a. Since the horizontal size corresponding to $\theta = \pm 50°$ is much larger than the thickness of the

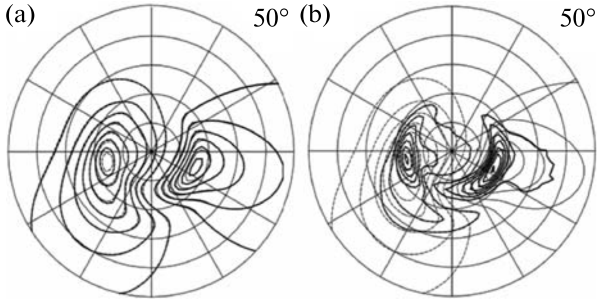

Fig. 3. Distributions of the vertical component of the electric field under quiet geomagnetic conditions (a) (thin lines) on the ground with an interval of $\delta E_r = 1.85$ mV m^{-1} and (thick lines) at an altitude of 40 km with an interval of $\delta E_r = 1.2$ mV m^{-1} and (b) (thick lines) at an altitude of 60 km with an interval of $\delta E_r = 0.15$ mV m^{-1} and (thin lines) the results of the one-dimensional model with an interval of $\delta E_r = 0.05$ mV m^{-1}.

atmospheric layer H, the vertical size is magnified by a factor of 20. The ionospheric potential is specified at an altitude of $H = 80$ km. The calculations show that the presented plot remains almost unchanged when H changes by ± 10 km.

Figure 3a shows the distributions of the vertical component of the electric field on the ground and at an altitude of 40 km. They differ from each other primarily in scale and are close to the results of the one-dimensional model, which provides the contours in Fig. 2a, but with the corresponding scaling of the interval between the contours. For example, an interval of $\delta V = 5$ kV in Fig. 2a is transformed to an interval of $\delta E_r = 1.2$ mV m^{-1} at an altitude of 40 km and to an interval of $\delta E_r = 1.85$ mV m^{-1} on the ground. These intervals between the contours were taken in Fig. 3a.

Figure 3b shows the vertical component of the electric field at an altitude of 60 km. This distribution is not similar to that obtained with the one-dimensional model. In particular, the fields near the maxima are tripled and, for this reason, the tripled interval between the contours is taken.

4. CONCLUSIONS

When the potential difference in the ionosphere is about 60 kV, the vertical field on the ground is 13 V m^{-1}, which is about one tenth of the mean field on the ground. According to the taken conductivity model, this mean field is 110 V m^{-1}, if the potential difference between the ground and ionosphere is 300 kV.

The analysis of the constructed solutions of the atmospheric conductivity shows that the electric field in the lower atmosphere can be obtained in the one-dimensional model, if the fields with horizontal sizes of no less than 100 km are considered. A three-dimensional conductivity model should be used for the upper atmosphere. The developed numerical method makes it possible to take into account the spatial inhomogeneities of the conductivity and ground relief.

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